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Optimization of Linear Quadratic Regulator with Tracking Applied to Autonomous Underwater Vehicle (AUV) Using Cuckoo Search

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Abstract: An Autonomous Underwater Vehicle (AUV) is used for exploring marine resources. The AUV has a control system for the surge, sway, or heave position and roll, pitch, or yaw angle. Tracking problems can be solved by using a controller designed using the LQR (Linear Quadratic Regulator). In optimal control for tracking problems using the LQR, the performance index is used as the objective function. The value of objective function depends on weighted matrices and, in general, the weighted matrices are determined by trial and error. In this research, the optimization of weighted matrices will be approached by heuristic methods such as Cuckoo Search (CS). CS simulates the reproduction strategy of cuckoo birds. The nests in CS represent weighted matrices in the LQR and the fitness function represents the performance index. Based on the simulation, the CS algorithm can find optimal weighted matrices in the LQR for the tracking problems. Furthermore, the solution of state and the optimal control can be obtained.

Keywords: *linear quadratic regulator; autonomous underwater vehicle; optimization; Cuckoo search.*

Mathematics Subject Classification (2010): 93C05, 93C15.

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1 Introduction

Compared to the whole territory of Indonesia, around seventy percent are in the form of sea. Thus, Indonesia has plenty of marine resources [1, 11]. For exploring marine resources, it is a necessity to have Autonomous Underwater Vehicles (AUV) together with their control [13, 14]. An AUV has three linear motions (surge, sway, heave positions) and three angular rotations (roll, pitch, yaw angle). Both angle and position ought to be controlled for maintaing a stable AUV. The controls for the AUV are rudder (for controlling surge and roll) and fin (for controlling sway, heave, pitch, and yaw) [5].

Control design for tracking problems by using the LQR (Linear Quadratic Regulator) has been developed in many applications. In optimal control for tracking problems using the LQR, the performance index is defined as the objective function. The value of objective function depends on weighted matrices and, generally, weighted matrices are determined by trial and error.

In the previous studies, the optimizations of weighted matrices in the optimal control model have been researched, among which are the optimization of weighted matrices in the optimal control of disease spread [4, 6], optimal control of inverted pendulum [3], fuzzy model for multimachine power systems [15], sliding mode control for wind energy conversion systems [16], PID for quadrotor performance [17]. In this research, the optimization of weighted matrices uses one of the heuristic methods called Cuckoo Search (CS).

The Cuckoo Search (CS) algorithm was proposed in [7, 8]. It simulates the reproduction strategy of cuckoo birds. They lay their eggs in the other bird's nest so that when the eggs are hatched, their chicks are fed by the other birds. Sometimes they remove existing eggs of the host nest in order to have higher probability of hatching their own eggs. Some species of cuckoo birds are specialized to mimic the pattern and color of the eggs of host birds so that the host birds could not recognize their eggs which have higher probability of hatching.

The contributions of this paper are as follows. In this paper, we use the CS algorithm to compute the weighted matrices that will be used in the LQR. After the optimized weighted matrices are obtained using CS, the weighted matrices will be used to compute the solution of state and optimal control for tracking problems by using the LQR. In the LQR, the performance index is defined as the objective function. In the simulations, a sinusoidal signal is used as the reference for surge position, sway position, heave position, roll angle, pitch angle, and yaw angle. Then we compare the reference and the trajectory generated by the closed-loop system.

2 Mathematical Modelling of AUV

In this section, first we derive the mathematical model of the AUV from the equation of motions. Then we extend the original model so that it can be used to solve the problem discussed in this paper.

2.1 State space model of AUV

In this subsection, we develop the mathematical model of the AUV from the equation of motions. The specifications and profile of the AUV used in this research are shown in Table 1 and Figure 1. As mentioned in the Introduction, the AUV has six degrees of freedom (6-DOF), that is, surge, sway, heave, roll, pitch and yaw [12]. The following

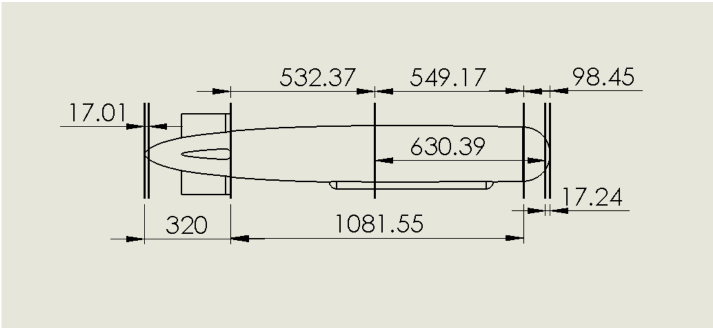


Figure 1: Profile of UNUSAITS AUV [13].

equations describe the dynamics of each motion for the AUV. The dynamics are described in the form of ordinary differential equations.

The dynamics of surge is

$$m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] = X_{|u|u}u|u| + X_{\dot{u}}\dot{u} + X_{wq}wq + X_{qq}qq + X_{vr}vr + X_{rr}rr + X_{prop}. \quad (1)$$

The dynamics of sway is

$$m[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(pq + \dot{r})] = Y_{res} + Y_{|v|v}v|v| + Y_{r|r|}r|r| + Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{ur}ur + Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r. \quad (2)$$

The dynamics of heave is

$$m[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] = Z_{res} + Z_{|w|w}w|w| + Z_{q|q|}q|q| + Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{uq}uq + Z_{vp}vp + Z_{rp}rp + Z_{uw}uw + Z_{uu\delta_s}u^2\delta_s. \quad (3)$$

The dynamics of roll is

$$I_x\dot{p} + (I_z - I_y)qr + m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] = K_{res} + K_{p|p|}p|p| + K_{\dot{p}}\dot{p} + K_{prop}. \quad (4)$$

The dynamics of pitch is

$$I_y\dot{q} + (I_x - I_z)rp + m[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)] = M_{res} + M_{w|w|}w|w| + M_{q|q|}q|q| + M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{uq}uq + M_{vp}vp + M_{rp}rp + M_{uw}uw + M_{uu\delta_s}u^2\delta_s. \quad (5)$$

The dynamics of yaw is

$$I_z\dot{r} + (I_y - I_z)pq + m[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] = N_{res} + N_{v|v|}v|v| + N_{r|r|}r|r| + N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_{ur}ur + N_{wp}wp + N_{pq}pq + N_{uv}uv + N_{uu\delta_r}u^2\delta_r. \quad (6)$$

The parameters in (1)-(6) are as follows: m is the mass of the AUV, I_x, I_y, I_z are the moment of inertia at the x-axis, y-axis, and z-

Weight	16 Kg
Length	1500 mm
Diameter	200 mm
Controller	Ardupilot Mega 2.0
Communication	Wireless Xbee 2.4 GHz
Camera	TTL Camera
Battery	Li-Pro 11.8 V
Propulsion	12V DC Motor
Propeller	3 Blades OD : 50 mm
Speed	3.1 knots (1.5 m/s)
Maximum depth	8 m

Table 1: Specification of UNUSAITS AUV [13].

axis, respectively, x_G, y_G, z_G are the longitudinal, athwart, and vertical position of the center of gravity, respectively. The others are [5]:

X : surge force	x : surge position	u : surge velocity
Y : sway force	y : sway position	v : sway velocity
Z : heave force	z : heave position	w : heave velocity
K : roll moment	ϕ : roll angle	p : roll rate
M : pitch moment	θ : pitch angle	q : pitch rate
N : yaw moment	ψ : yaw angle	r : yaw rate

Then we linearize the model around an equilibrium point by using the Jacobian method. The linearized state space model can be expressed as follows:

$$\dot{X}_v = A_v X_v + B_v U_v, \tag{7}$$

$$Y_v = C_v X_v. \tag{8}$$

For the AUV model, the surge velocity u, sway velocity v, heave velocity w, roll rate p, pitch rate q, and yaw rate r can be written in the general form of the state space model as follows:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = A_v \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix} + B_v \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{bmatrix}, \quad \begin{bmatrix} Y_{v1} \\ Y_{v2} \\ Y_{v3} \\ Y_{v4} \\ Y_{v5} \\ Y_{v6} \end{bmatrix} = C_v \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}, \tag{9}$$

where the matrices A_v, B_v, C_v are

$$A_v = \dots \tag{10}$$

$$B_v = \dots \tag{11}$$

$$C_v = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{12}$$

2.2 Development of state space model of AUV

State space model (7)-(8) can produce a solution of surge velocity u, sway velocity v, heave velocity w, roll rate p, pitch rate q, and yaw rate r. We will develop a state space model in order to produce a solution of surge position x, sway position y, heave position z, roll angle ϕ, pitch angle θ, and yaw angle ψ.

Assume that velocity is the first derivative of position and rate is the first derivative of angle, then we obtain

$$\dot{x} = u, \quad \dot{y} = v, \quad \dot{z} = w, \quad \dot{\phi} = p, \quad \dot{\theta} = q, \quad \dot{\psi} = r. \tag{13}$$

There are six controls U_v in the surge velocity, sway velocity, heave velocity, roll rate, pitch rate, and yaw rate:

$$U_v = \begin{bmatrix} \delta_1 & \delta_2 & \delta_3 & \delta_4 & \delta_5 & \delta_6 \end{bmatrix}^T. \tag{14}$$

There is no control for the surge position, sway position, heave position, roll angle, pitch angle, and yaw angle. In the output matrix, we only observe the surge position, sway position, heave position, roll angle, pitch angle, and yaw angle to be minimized in the performance index of the LQR optimal control. Therefore, the state space model (9) can be extended to become the following equations:

$$\begin{bmatrix} \dot{X}_v \\ \dot{X}_p \end{bmatrix} = \begin{bmatrix} A_v & \bar{0} \\ I & \bar{0} \end{bmatrix} \begin{bmatrix} X_v \\ X_p \end{bmatrix} + \begin{bmatrix} B_v \\ \bar{0} \end{bmatrix} U_v, \tag{15}$$

$$\begin{bmatrix} Y_v \\ Y_p \end{bmatrix} = \begin{bmatrix} \bar{0} & \bar{0} \\ \bar{0} & I \end{bmatrix} \begin{bmatrix} X_v \\ X_p \end{bmatrix}, \tag{16}$$

where $X_p = \begin{bmatrix} x & y & z & \phi & \theta & \psi \end{bmatrix}^T$, I is an identity matrix, and $\bar{0}$ is the matrix whose all elements are zero.

In state space model (15) and (16), the size of matrix $\begin{bmatrix} A_v & \bar{0} \\ I & \bar{0} \end{bmatrix}$ is 12×12, the size of input matrix $\begin{bmatrix} B_v \\ \bar{0} \end{bmatrix}$ is 12×6, and the size of output matrix $\begin{bmatrix} \bar{0} & \bar{0} \\ \bar{0} & I \end{bmatrix}$ is 12×12.

3 Linear Quadratic Regulator

The Linear Quadratic Regulator (LQR) will be used in determining the optimal control of AUV. The optimal control problem is to find an optimal control $u^*(t)$, such that the corresponding trajectory $x^*(t)$ minimizes the given performance index J [10].

3.1 Tracking problems for discrete-time systems using LQR

In this research, we describe the tracking problems and its solution by using the LQR. The objective of tracking problems is that the output of the system follows a desired trajectory r that minimizes a performance index J. The discrete-time model for tracking problems by using the LQR is defined as follows.

The state $\dot{x}(t)$ and output equation $y(t)$ in discrete time can be constructed in the following equations:

$$x_{k+1} = Ax_k + Bu_k, \tag{17}$$

$$y_k = Cx_k, \tag{18}$$

and the performance index that is defined as the objective function is defined as follows:

$$J = \frac{1}{2} e_N^T P e_N + \frac{1}{2} \sum_{k=0}^{N-1} (e_k^T Q e_k + u_k^T R u_k), \tag{19}$$

where $e_N = Cx_N - r_N$ and $e_k = Cx_k - r_k$.

The error weighted matrices $P \geq 0$ and $Q \geq 0$ must be symmetric and positive semidefinite, whereas the control weighted matrix $R > 0$ must be symmetric and positive definite.

3.2 Tracking algorithm by using LQR

The tracking algorithm and computation by using the LQR for discrete-time systems are as follows [2]:

1. Compute K_k, S_k, v_k, K_k^v backward with the final conditions $S_N = C^T PC$ and $v_N = C^T P r_N$.
   For $k = N-1, N-2, \ldots, 0$ do
   $$K_k = (B^T S_{k+1} B + R)^{-1} B^T S_{k+1} A, \tag{20}$$
   $$S_k = C^T QC + A^T S_{k+1}(A - BK_k), \tag{21}$$
   $$v_k = (A - BK_k)^T v_{k+1} + C^T Q r_k, \tag{22}$$
   $$K_k^v = (B^T S_{k+1} B + R)^{-1} B^T, \tag{23}$$
   End For
2. Compute x_k forward with the initial conditions x_0.
   For $k = 1, 2, \ldots, N-1$ do
   $$x_{k+1} = Ax_k + B(-K_k x_k + K_k^v v_{k+1}), \tag{24}$$
   End For
3. Compute the optimal control u_k:
   $$u_k = -K_k x_k + K_k^v v_{k+1}. \tag{25}$$
4. Compute the performance index J as the objective function:
   $$J = \frac{1}{2} e_N^T P e_N + \frac{1}{2} \sum_{k=0}^{N-1} (e_k^T Q e_k + u_k^T R u_k), \tag{26}$$
   where $e_N = Cx_N - r_N$ and $e_k = Cx_k - r_k$.

4 Cuckoo Search

The Cuckoo Search (CS) algorithm was proposed in [7]. It imitates the reproduction strategy of cuckoo birds. The cuckoo birds lay their eggs in the other bird's nest. So, when the eggs are hatched, their chicks are fed by the other birds. Sometimes they remove existing eggs of the host nest in order to increase the probability of hatching their own eggs. Some species of cuckoo birds are specialized to mimic the pattern and color of the host bird's eggs, so the host birds could not recognize their eggs, meaning it ensures high probability of the hatching [8].

4.1 Behavior of the cuckoo

Based on the behavior of cuckoo birds, the CS algorithm uses three idealized rules:

1. Each cuckoo lays one egg at a time and dumps it in a randomly selected nest.
2. The best nest with high quality eggs will be carried over to the next generation.
3. The number of available host nests is fixed and a host bird can discover a strange egg with a probability $p_a \in [0, 1]$. In this case, the host bird can either throw the egg away or abandon the nest to build a completely new nest in a new location.

In the original CS, there is no distinction between an egg, a nest, or a cuckoo. Each nest corresponds to one egg which also represents one cuckoo.

4.2 Cuckoo Search algorithm on LQR

In tracking problems by using the LQR, there are three weighted matrices which will be optimized, i.e., $P \geq 0$, $Q \geq 0$ which must be symmetric and positive semidefinite, and $R > 0$ which must be symmetric and positive definite.

We assume $P = \bar{0}$. Because of the only surge position, sway position, heave position, roll angle, pitch angle, and yaw angle which will be minimized, one has $Q = \begin{bmatrix} \bar{0} & \bar{0} \\ \bar{0} & Q_x \end{bmatrix}$ with Q_x being

$$Q_x = \begin{bmatrix} q_7 & 0 & 0 & 0 & 0 & 0 \\ 0 & q_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & q_9 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & q_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & q_{12} \end{bmatrix}, \tag{27}$$

where $q_i > 0$, $i = 7, 8, \ldots, 12$.

Because there are six controls U_v in the surge velocity, sway velocity, heave velocity, roll rate, pitch rate, and yaw rate

$$U_v = \begin{bmatrix} \delta_1 & \delta_2 & \delta_3 & \delta_4 & \delta_5 & \delta_6 \end{bmatrix}^T, \tag{28}$$

$R > 0$ can be constructed as follows:

$$R = \begin{bmatrix} r_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_6 \end{bmatrix}, \tag{29}$$

where $r_i > 0$, $i = 1, 2, \ldots, 6$.

Therefore, the representation of decision variable which will be used in CS as the host nest is

$$X = \begin{bmatrix} q_7 & q_8 & q_9 & \ldots & q_{12} & r_1 & r_2 & r_3 & \ldots & r_6 \end{bmatrix}, \tag{30}$$

where the fitness function is defined as the performance index J in (26). Based on the behavior of cuckoo birds, the CS algorithm for optimizing weighted matrices in the LQR with tracking can be designed as follows.

Generate the initial population of host nests x_i, $i = 1, 2, \ldots, maxpop$, randomly. Each of them represents a candidate solution to the optimization problem with the objective function $f(x)$ [9].

For $t = 1, 2, \ldots, tmax$

1. Calculate the global random walk and generate a new nest x_i^{t+1} using the Levy flight:

$$x_i^{t+1} = x_i^t + \alpha \otimes Levy(s, \lambda),$$

where $\alpha > 0$ is the step size scaling factor. The search steps in terms of random $Levy(s, \lambda)$ should be drawn from the Levy distribution. In addition, $\otimes$ denotes the entry-wise multiplication.

$$Levy \sim \frac{\lambda \Gamma(\lambda) \sin\left(\frac{\lambda}{2}\pi\right)}{\pi} \frac{1}{s^{1+\lambda}}, \quad s > 0.$$

The letter Γ represents the gamma function $\Gamma(z) = \int_0^\infty t^{z-1} e^{-z} dt$. If $z = k$ is a positive integer, then $\Gamma(k) = (k-1)!$.

2. Evaluate the fitness of x_i^{t+1}

3. Choose a new nest j randomly from $maxpop$ initial nests. If the fitness of x_i^{t+1} is better than x_i^t, replace j by x_i^{t+1}

4. Abandon some of the worst nests and build new ones. It depends on the discovery probability parameter p_a. Generate a uniformly distributed random number $\varepsilon \sim U(0, 1)$.
If $\varepsilon < p_a$
Create a new nest using the local random walk

$$x_i^{t+1} = x_i^t + \alpha \otimes H(p_a - \varepsilon) \otimes (x_j^t - x_k^t),$$

where $\alpha > 0$ and $H(p_a - \varepsilon)$ is a Heaviside function.
Evaluate the fitness of x_i^{t+1} and find the best one
End If

5. Update the best solution

End For

5 Simulation Results

Based on the AUV data, the state space model in (15) uses the following matrices A_v and B_v:

$$A_v = \begin{bmatrix} 0.098 & 0.032 & -0.179 & 0.119 & -0.037 & -0.148 \\ -0.001 & 0.098 & 0.077 & 0.083 & 0.039 & 0.026 \\ -0.089 & 0.030 & -0.066 & 0.008 & -0.113 & -0.128 \\ 0.139 & -0.027 & 0.019 & -0.988 & -0.371 & 0.139 \\ 0.047 & 0.004 & -0.032 & -0.993 & -0.469 & 0.031 \\ 0.139 & -0.114 & -0.054 & -0.879 & -0.411 & 0.119 \end{bmatrix},$$

$$B_v = \begin{bmatrix} 0.0177 & 0.0003 & -0.0003 & 0 & -0.0003 & 0.0003 \\ 0.0012 & 1.49\times10^{-5} & -1.80\times10^{-5} & -0.6070 & -1.80\times10^{-5} & 1.49\times10^{-5} \\ 0.0156 & 0.0002 & -0.0002 & 0.6070 & -0.0002 & 0.0002 \\ -0.0153 & -0.0002 & 0.0002 & 5.8592 & 0.0002 & -0.0002 \\ -0.0019 & -2.97\times10^{-5} & 0.0005 & 6.4357 & 0.0005 & -2.97\times10^{-5} \\ -0.0164 & 0.0002 & 0.0003 & 6.4357 & 0.0003 & 0.0002 \end{bmatrix}.$$

5.1 Cuckoo Search simulation on standard LQR without tracking

Tracking problems by using the LQR will be used in the AUV model for determining the solution of state as in (24) and the optimal control as in (25). There are weighted matrices whose elements will be optimized by the CS algorithm. In the CS simulation, the parameters used are:

- The number of nests = 10.
- Maximum iteration = 50.
- Discovery probability parameter $p_a = 0.5$.

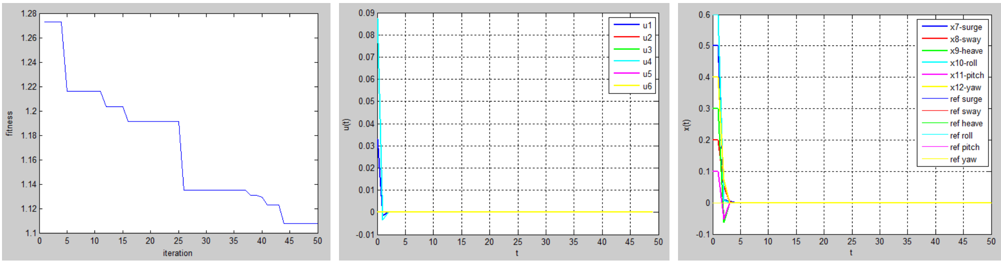
Figure 2(a) is an optimization process of the CS algorithm for optimizing the weighted matrices of LQR. First, there are some nests at a random position. At the optimization process using the Levy flight, by abandoning some of the worst nests and building new nests, the position of the best nest is found. The Optimal Performance Index as the fitness function is 1.1081 with the elements of weighted matrices being

$$\begin{aligned} X &= \begin{bmatrix} q_7 & q_8 & q_9 & \dots & q_{12} & r_1 & r_2 & r_3 & \dots & r_6 \end{bmatrix} \\ &= [1.0197 \quad 2.0534 \quad 1.0841 \quad 1.1568 \quad 1.8789 \quad 1.4099 \quad 0.0711 \quad 0.5264 \quad 0.2167 \quad \dots \\ &\qquad 0.3140 \quad 0.2686 \quad 0.8924]. \end{aligned}$$

After the optimal weighted matrices are obtained, they will be applied in the LQR simulation of the AUV model. Figure 2(b) is the optimal control of LQR, with the solutions of state as in Figure 2(c).

5.2 Cuckoo Search simulation on LQR with tracking

Tracking problems by using the LQR will be used in the AUV model for determining the solution of state as in (24) and the optimal control as in (25). In tracking problems by using the LQR, the reference used is $\sin t$, $t = 1, 2, \dots, T$ for each position and angle. There are weighted matrices whose elements will be optimized by the CS algorithm. In the CS simulation, the parameters used are:



(a) (b) (c)

Figure 2: (a) The CS optimization process (b) The optimal control of LQR (c) The solution of state of LQR.

- The number of nests = 10.
- Maximum iteration = 50.
- Discovery probability parameter $p_a = 0.5$.

Figure 3(a) is an optimization process of the CS algorithm for optimizing the weighted matrices of the LQR with tracking $\sin t$, $t = 1, 2, \ldots, T$, for the surge position and zero otherwise.

First, there are some nests at a random position. At the optimization process using the Levy flight, by abandoning some of the worst nests and building new nests, the position of the best nest is found. The Optimal Performance Index as the fitness function is 11.0361 with the elements of weighted matrices being

$$\begin{aligned} X &= \begin{bmatrix} q_7 & q_8 & q_9 & \ldots & q_{12} & r_1 & r_2 & r_3 & \ldots & r_6 \end{bmatrix} \\ &= [1.0387 \quad 1.4831 \quad 1.9183 \quad 1.6391 \quad 3.8579 \quad 2.3461 \quad 0.0003 \quad 0.0883 \quad 0.6869 \quad \ldots \\ &\qquad 0.4850 \quad 0.7080 \quad 0.0933]. \end{aligned}$$

After the optimal weighted matrices are obtained, they will be applied in tracking problems by using the LQR for the AUV model. Figure 3(b) is the optimal control of tracking problems by using the LQR on the surge position, with the solutions of state as in Figure 3(c). Figure 3(d) is the comparison between the solution of the surge position and its reference.

Figure 4(a) is an optimization process of the CS algorithm for optimizing the weighted matrices of tracking problems by using the LQR with reference $\sin t$, $t = 1, 2, \ldots, T$, for the sway position and zero otherwise. First, there are some nests at a random position. At the optimization process using the Levy flight, by abandoning some of the worst nests and building new nests, the position of the best nest is found. The Optimal Performance Index as the fitness function is 13.7676 with the elements of weighted matrices being

$$\begin{aligned} X &= \begin{bmatrix} q_7 & q_8 & q_9 & \ldots & q_{12} & r_1 & r_2 & r_3 & \ldots & r_6 \end{bmatrix} \\ &= [1.8594 \quad 1.0174 \quad 1.4118 \quad 1.4144 \quad 1.6101 \quad 1.2758 \quad 0.8354 \quad 0.2502 \quad 0.5636 \quad \ldots \\ &\quad 0.3001 \quad 0.0801 \quad 0.5326]. \end{aligned}$$

After the optimal weighted matrices are obtained, they will be applied in the simulation of tracking problems by using the LQR for the AUV model. Figure 4(b) is the

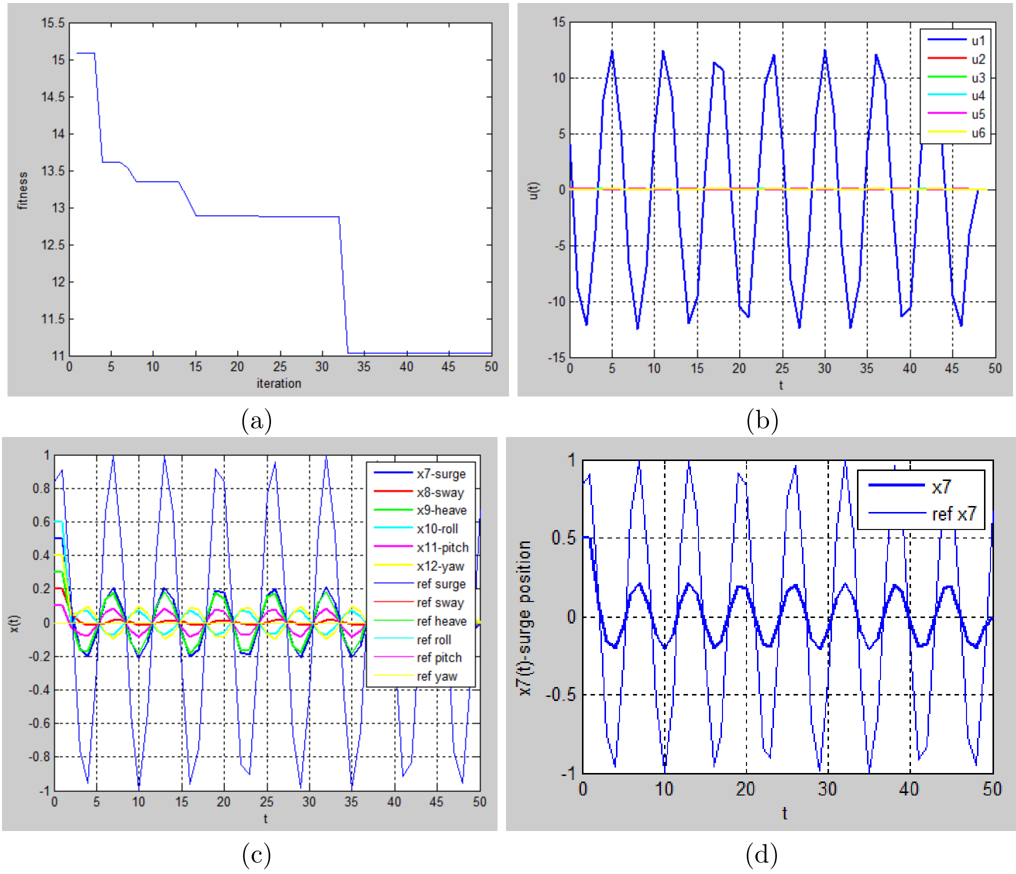


Figure 3: (a) The CS optimization process (b) The optimal control of tracking problems by using the LQR on the surge position (c) The solution of state of tracking problems by using the LQR on the surge position (d) Comparison between the reference and the surge position.

optimal control of tracking problems by using the LQR on the sway position, with the solutions of state as in Figure 4(c). Figure 4(d) is the comparison between the solution of the sway position and its reference.

Figure 5(a) is an optimization process of the CS algorithm for optimizing the weighted matrices of tracking problems by using the LQR with reference $\sin t$, $t = 1, 2, \ldots, T$, for the heave position and zero otherwise. First, there are some nests at a random position. At the optimization process using the Levy flight, by abandoning some of the worst nests and building new nests, the position of the best nest is found. The Optimal Performance Index as the fitness function is 12.3018 with the elements of weighted matrices being

$$\begin{aligned} X &= \begin{bmatrix} q_7 & q_8 & q_9 & \ldots & q_{12} & r_1 & r_2 & r_3 & \ldots & r_6 \end{bmatrix} \\ &= [2.8670 \quad 2.5503 \quad 1.0348 \quad 1.1602 \quad 3.4523 \quad 1.2199 \quad 0.0003 \quad 0.0255 \quad 0.0590 \quad \ldots \\ &\qquad 0.2296 \quad 0.2725 \quad 0.0403]. \end{aligned}$$

After the optimal weighted matrices are obtained, they will be applied in the simulation of tracking problems by using the LQR for the AUV model. Figure 5(b) is the

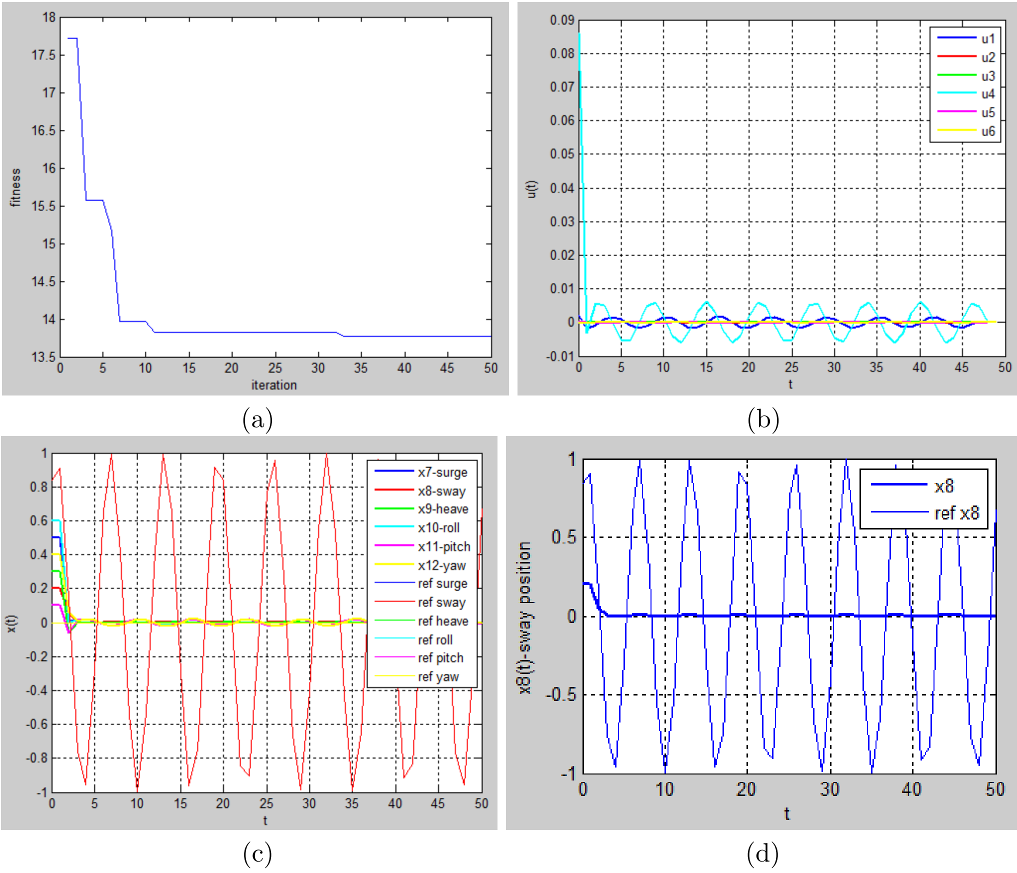


Figure 4: (a) The CS optimization process (b) The optimal control of tracking problems by using the LQR on the sway position (c) The solution of state of tracking problems by using the LQR on the sway position (d) Comparison between the reference and the sway position.

optimal control of tracking problems by using the LQR on the heave position, with the solutions of state as in Figure 5(c). Figure 5(d) is the comparison between the solution of the heave position and its reference.

Figure 6(a) is an optimization process of the CS algorithm for optimizing the weighted matrices of tracking problems by using the LQR with reference $\sin t$, $t = 1, 2, \ldots, T$, for the roll angle and zero otherwise. First, there are some nests at a random position. At the optimization process using the Levy flight, by abandoning some of the worst nests and building new nests, the position of the best nest is found. The Optimal Performance Index as the fitness function is 9.9797 with the elements of weighted matrices being

$$\begin{aligned} X &= \begin{bmatrix} q_7 & q_8 & q_9 & \ldots & q_{12} & r_1 & r_2 & r_3 & \ldots & r_6 \end{bmatrix} \\ &= [2.2584 \quad 1.3483 \quad 1.2542 \quad 1.0173 \quad 1.1281 \quad 1.1688 \quad 0.7712 \quad 0.8242 \quad 0.0241 \quad \ldots \\ &\qquad 0.6006 \quad 0.6243 \quad 0.0782]. \end{aligned}$$

After the optimal weighted matrices are obtained, they will be applied in the simulation of tracking problems by using the LQR for the AUV model. Figure 6(b) is the

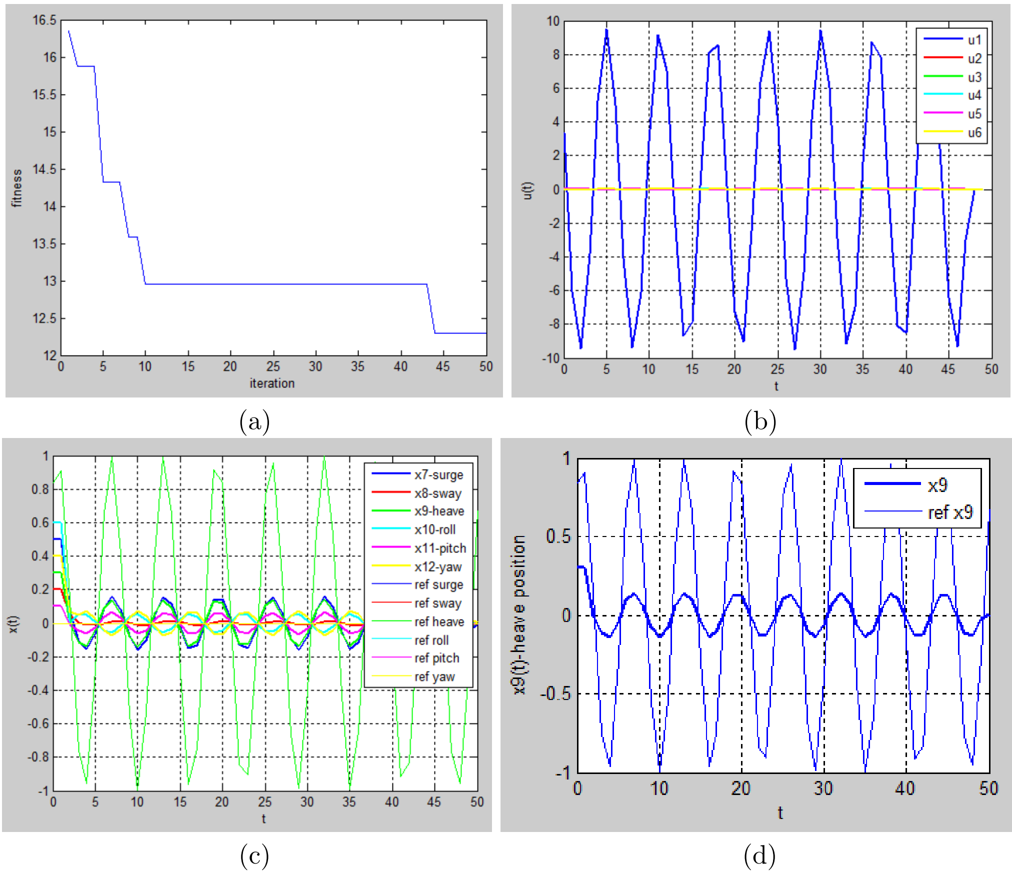


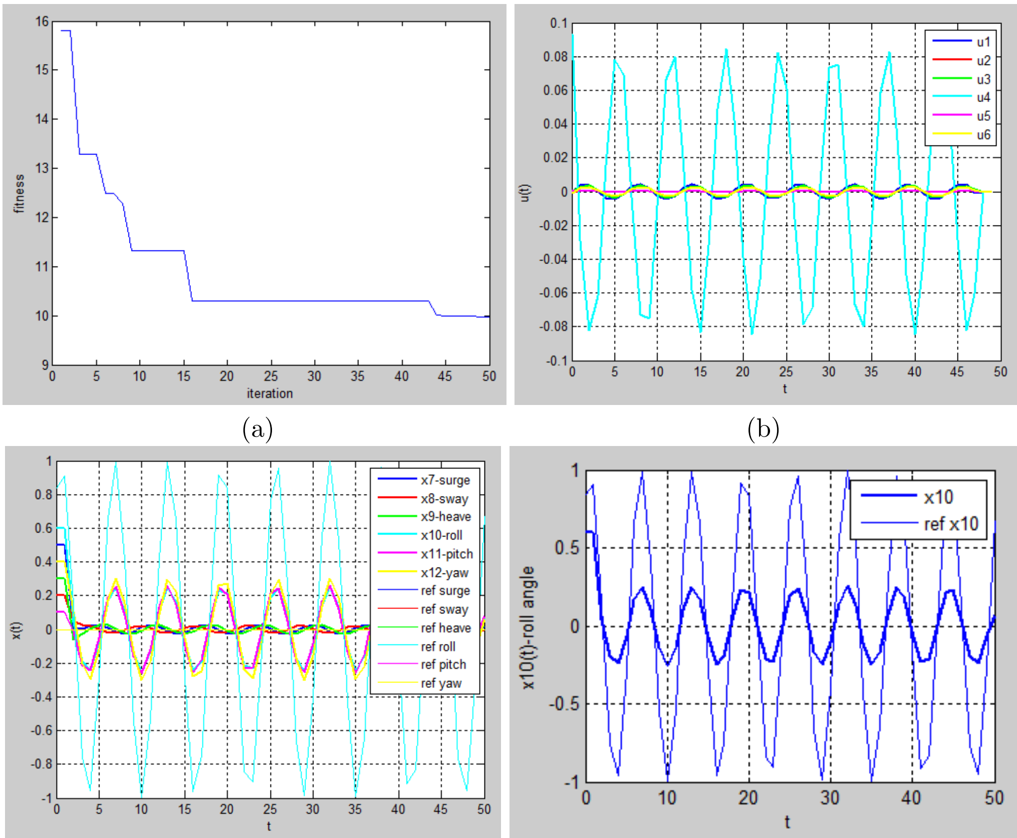
Figure 5: (a) The CS optimization process (b) The optimal control of tracking problems by using the LQR on the heave position (c) The solution of state of tracking problems by using the LQR on the heave position (d) Comparison between the reference and the heave position.

optimal control of tracking problems by using the LQR on the roll angle, with the solutions of state as in Figure 6(c). Figure 6(d) is the comparison between the solution of the roll angle and its reference.

Figure 7(a) is an optimization process of the CS algorithm for optimizing the weighted matrices of the LQR with tracking $\sin t$, $t = 1, 2, \ldots, T$, for the pitch angle and zero otherwise. First, there are some nests at a random position. At the optimization process using the Levy flight, by abandoning some of the worst nests and building new nests, the position of the best nest is found. The Optimal Performance Index as the fitness function is 10.2451 with the elements of weighted matrices being

$$\begin{aligned} X &= \begin{bmatrix} q_7 & q_8 & q_9 & \ldots & q_{12} & r_1 & r_2 & r_3 & \ldots & r_6 \end{bmatrix} \\ &= [2.0025 \quad 2.0480 \quad 2.2970 \quad 1.0217 \quad 1.0020 \quad 1.0005 \quad 0.4297 \quad 0.5129 \quad 0.7946 \quad \ldots \\ &\qquad 0.2648 \quad 0.3575 \quad 0.9104]. \end{aligned}$$

After the optimal weighted matrices are obtained, they will be applied in the simulation of tracking problems by using the LQR for the AUV model. Figure 7(b) is the



(a) (b)

(c) (d)

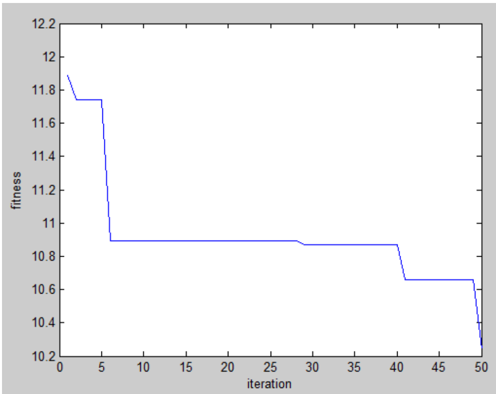
Figure 6: (a) The CS optimization process (b) The optimal control of tracking problems by using the LQR on the roll angle (c) The solution of state of tracking problems by using the LQR on the roll angle (d) Comparison between the reference and the roll angle.

optimal control of the LQR with tracking on the pitch angle, with the solutions of state as in Figure 7(c). Figure 7(d) is the comparison between the solution of the pitch angle and its reference.

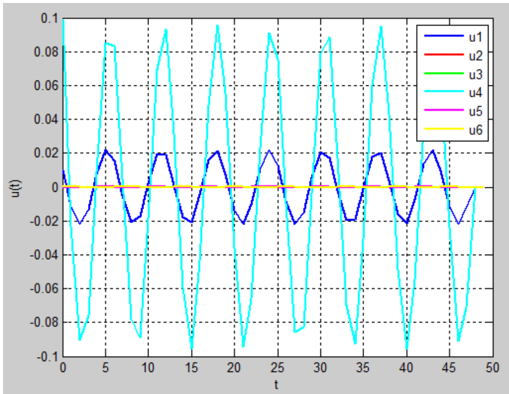
Figure 8(a) is an optimization process of the CS algorithm for optimizing the weighted matrices of tracking problems by using the LQR with reference $\sin t$, $t = 1, 2, \ldots, T$, for the yaw angle and zero otherwise. First, there are some nests at a random position. At the optimization process using the Levy flight, by abandoning some of the worst nests and building new nests, the position of the best nest is found. The Optimal Performance Index as the fitness function is 8.7282 with the elements of weighted matrices being

$$\begin{aligned} X &= \begin{bmatrix} q_7 & q_8 & q_9 & \ldots & q_{12} & r_1 & r_2 & r_3 & \ldots & r_6 \end{bmatrix} \\ &= [1.1022 \quad 1.5842 \quad 1.1603 \quad 1.1180 \quad 1.1842 \quad 1.0430 \quad 0.3255 \quad 0.0185 \quad 0.6339 \quad \ldots \\ &\qquad 0.0825 \quad 0.9782 \quad 0.6808]. \end{aligned}$$

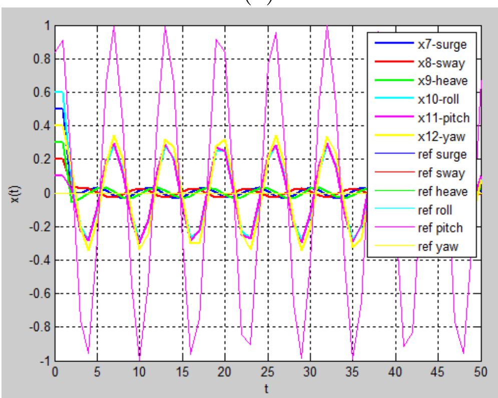
After the optimal weighted matrices are obtained, they will be applied in the simu-



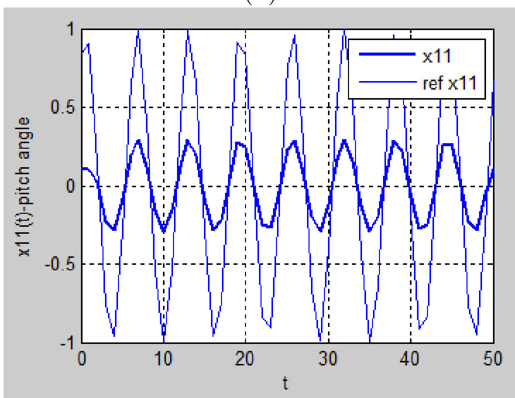
(a)



(b)



(c)



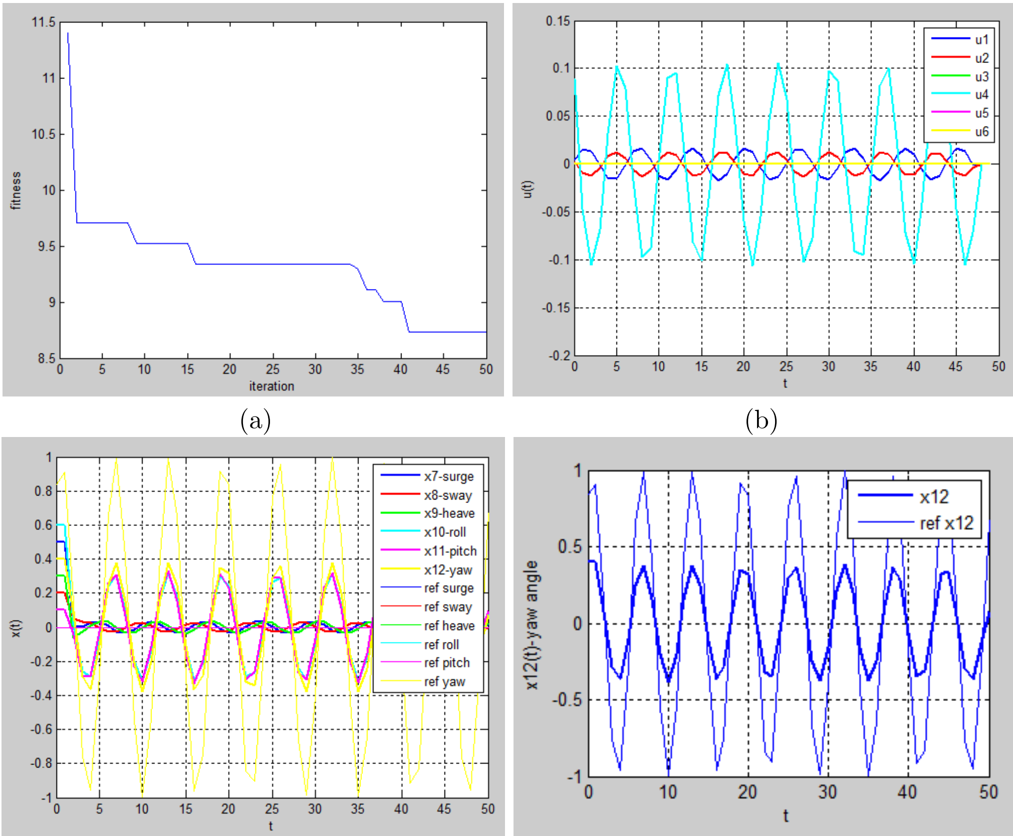
(d)

Figure 7: (a) The CS optimization process (b) The optimal control of tracking problems by using the LQR on the pitch angle (c) The solution of state of tracking problems by using the LQR on the pitch angle (d) Comparison between the reference and the pitch angle.

lation of tracking problems by using the LQR for the AUV model. Figure 8(b) is the optimal control of tracking problems by using the LQR on the yaw angle, with the solutions of state as in Figure 8(c). Figure 8(d) is the comparison between the solution of the yaw angle and its reference.

6 Conclusion

Tracking problems by using the LQR have been applied to the AUV model for determining the solution of state consisting of the surge position, sway position, heave position, roll angle, sway angle and yaw angle and the optimal control. In the tracking problems by using the LQR, there are weighted matrices which need to be optimized. The Cuckoo Search (CS) algorithm can be applied in optimization to obtain the weighted matrices. Based on simulation, the CS algorithm can find optimal weighted matrices in tracking problems by using the LQR. Furthermore, the solution of state and the optimal control can be obtained.



(a) (b)

(c) (d)

Figure 8: (a) The CS optimization process (b) The optimal control of tracking problems by using the LQR on the yaw angle (c) The solution of state of tracking problems by using the LQR on the yaw angle (d) Comparison between the reference and the yaw angle.

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