Nonlinear Dynamics and Systems Theory, **20** (2) (2020) 191–202

The Finite Element Method for Nonlinear Nonstandard Volterra Integral Equations

M. Khumalo * and A. Dlamini

University of South Africa, P.O. Box 392, UNISA 0003, Republic of South Africa

Received: November 19, 2019; Revised: March 15, 2020

Abstract: In this work, we look at the implementation of the finite element method to a nonlinear (nonstandard) Volterra integral equation. We consider the Galerkin approach, where we choose the weight function in such a way that it takes the form of the approximate solution. We work on a uniform mesh and choose the Lagrange polynomials as basis functions. We consider the error analysis of the method. We look at a specific example to illustrate the implementation of the finite element method. Finally, we consider the estimated rate of convergence.

Keywords: *Volterra integral equations; finite element method; Galerkin approach.*

Mathematics Subject Classification (2010): 45D05, 65R20, 65N30.

1 Introduction

In this paper, we consider the nonlinear Volterra integral equation of the second kind

$$u(x) = \sum_{m=1}^{r} b_m \left(g_m(x) + \int_0^x k_m(x,y)u(y)dy \right)^m , \qquad x \in [0, L], \tag{1}$$

where $r \in \mathbb{N}, r \geq 2$, $b \in \mathbb{R}$, $g : [0, L] \to \mathbb{R}$ and $k : [0, L] \times [0, L] \to \mathbb{R}$ are continuous functions. The unknown function $u(x) \in C[0, L]$.

In essence, (1) is nonstandard in that in its simplest form, it has the structure

$$u = \sum_{m=1}^{r} b_m \left(g_m + W_m u\right)^m , \qquad x \in [0, L],$$

* Corresponding author: mailto:khumam@unisa.ac.za