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Some Problems of Attitude Dynamics and Control of a Rigid Body

to the 90th Birthday of Professor V. I. Zubov

A. Yu. Aleksandrov [1*], A. A. Martynyuk [2] and A. A. Tikhonov [1]

[1] *Saint Petersburg State University, 7-9 Universitetskaya Nab., St. Petersburg, 199034, Russia*
[2] *S. P. Timoshenko Institute of Mechanics, National Academy of Sciences of Ukraine, 3 Nesterov Str., 03057, Kyiv-57, Ukraine*

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Abstract: In the present paper, Vladimir Zubov's results on the problems of analysis and control of rotation motion of a rigid body are surveyed together with their developments and extensions.

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Mathematics Subject Classification (2010): 70E17, 70E50, 70Q05.

1 Introduction

The outstanding Russian mathematician and mechanical engineer Vladimir Ivanovich Zubov (1930–2000) made an invaluable contribution to the development of Stability Theory and Control Theory.

V. I. Zubov was born on April 14, 1930 in Kashira town, Moscow region, Russia. In 1945, he finished secondary school. At the age of 14, Vladimir was wounded by a hand grenade exploded accidentally and soon failed eyesight. In 1949, he finished the Leningrad special school for blind and visually impaired children and entered the Mathematical and Mechanical Faculty of the Leningrad State University. In 1953, after graduating with honors, he joined the University faculty and since then his career was inseparably associated with the Leningrad (now, Saint Petersburg) State University.

In 1955, V. I. Zubov defended his PhD thesis "Boundaries of the Asymptotic Stability Domain" in which he proved the theorem on the asymptotic stability domain. This result is now known as *Zubov's theorem.*

* Corresponding author: mailto:a.u.aleksandrov@spbu.ru

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Further Zubov's activities involved both pure fundamental investigations and solution of applied real-life problems in several fields — from spacecraft to ship control.

In 1969, the Faculty of Applied Mathematics and Control Processes was founded at the Leningrad State University with Vladimir Zubov's appointment as its first dean. Two years later, a Research Institute of Computational Mathematics and Control Processes was set up by the USSR Government. Zubov became its brains-and-heart. In particular, he headed the projects on the design, development and operation of systems of self-guided winged missiles, and tactical schemes construction for the USSR Navy to oppose aircraft carriers of the potential enemy.

Zubov's scientific activities were surveyed in the paper [1] dedicated to the 80th anniversary of his birth. Zubov's works on the problem of stability by nonlinear approximation are surveyed in [2]. In the present contribution, we focus on Zubov's results on the problems of analysis and control of rotation motion of a rigid body together with their development and extensions in the works of his disciples and followers.

2 A Survey of Zubov's Results

2.1 Investigation of rotation motion of a rigid body

V. I. Zubov succeeded to make essential contributions to the domains of analytical mechanics that had been exhaustively investigated by predecessors, and where it was hard to expect an original result. In the monographs [3–5], he examined the dynamics of the rotational motion of a rigid body around a fixed point in the following three directions:

(i) The complete theory of the motion of a rigid body in the Euler-Poinsot case.

(ii) The complete theory of the motion of a rigid body in the case of Lagrange-Poisson.

(iii) The theory of motion of a heavy solid in the general case in a constant uniform field of gravity.

These problems are considered fundamental in theoretical mechanics. Just a few problems of nonlinear dynamics admit a solution by quadratures; nevertheless, any of such a solution always attracts the interest of researchers. Until now, works devoted to the search for integrable particular cases in the dynamics of a rigid body continue to appear. Most of them discuss purely speculative constructions, the practical significance of which, as a rule, is not discussed. Unlike the background of these works, the works of V. I. Zubov on classical solid mechanics are theoretically elegant but, on the other hand, application-oriented. It should be noted that V.I. Zubov did not concern much about the problems of existence and uniqueness of solutions to the Euler equations of the rigid body motion; his interests were focused on the practical questions relevant to the control of the body's attitude. In particular, he is interested in the qualitative behavior of the spin axis of the body. V.I. Zubov introduced the notion of stability of a rigid body with respect to orientation (see [4]).

Definition 2.1 A body with a fixed point O is stable with respect to orientation if its main axis Oz remains all the time in the half-space bounded by the plane orthogonal to the momentum vector and passing through the point O.

In the Euler-Poinsot case, the equations of motion of a rigid body have the form

$$A\dot{p} + (C - B)qr = 0, \quad B\dot{q} + (A - C)pr = 0, \quad C\dot{r} + (B - A)pq = 0. \tag{1}$$

Here A, B, C are the principal central moments of inertia of the body, p, q, r are the projections of the body angular velocity on the principal axes of inertia Ox, Oy, Oz.

V. I. Zubov proved that the functions $v_1 = \gamma r^2 - \alpha r^2$, $v_2 = \gamma q^2 - \beta r^2$ with $\alpha = (C-B)/A$, $\beta = (A-C)/B$, $\gamma = (B-A)/C$ are the first integrals for the system (1). In terms of v_1 and v_2, he has formulated the following theorem [4].

Theorem 2.1 *A body is stable with respect to orientation if and only if the inequalities $\alpha\beta < 0$, $v_1 v_2 \leqslant 0$ hold.*

In the Lagrange case, the motions of a dynamically symmetric ($A = B$) rigid body with the mass m in a constant uniform field of gravity with the intensity g are described by the equations

$$A\dot{p} + (C-B)qr = -mgly, \quad B\dot{q} + (A-C)pr = mglx, \quad C\dot{r} = 0, \tag{2}$$

$$\dot{x} = ry - qz, \quad \dot{y} = -rx + pz, \quad \dot{z} = qx - py, \tag{3}$$

where x, y, z are the projections of the unit vector directed opposite to the gravity force on the axes Ox, Oy, Oz, and the mass centre (point G) has the coordinates $(0, 0, l)$ in the same reference frame. For this case, necessary and sufficient conditions for the stability of a body with respect to orientation were found as well [4].

V. I. Zubov has established that in the Euler and Lagrange cases, all the motions of a rigid body around a fixed point are periodic or almost periodic with the exception of the motions lying in a special integral manifold. He has determined the precise bounds of the nutational oscillations for the spin axis of a dynamically asymmetric rigid body freely rotating about a fixed point. Furthermore, he has determined stability and instability conditions of the rigid body motions with respect to the spatial orientation of axes [4, 5].

For the problem of the attitude motion of a heavy rigid body in a constant uniform field of gravity, the following equations were studied:

$$\begin{aligned} A\dot{p} + (C-B)qr &= mg(y_G z - z_G y), \\ B\dot{q} + (A-C)pr &= mg(z_G x - x_G z), \\ C\dot{r} + (B-A)pq &= mg(x_G y - y_G x), \end{aligned} \tag{4}$$

where x_G, y_G, z_G are the coordinates of the body mass centre in the coordinate system $Oxyz$. V.I. Zubov proved that any real solution of the Euler-Poinsot differential equations (3), (4) exists and is holomorphic in the strip of the complex plane that is symmetric with respect to the real axis. This solution can be converted into the series converging for all t [4].

The solution to the Darboux problem (the problem of determination of a rigid body attitude motion via given initial orientation and initial angular velocity) was also presented by V.I. Zubov in the form of a series converging for all t [3, 4]. The coefficients of this series are determined by recurrent formulas, which allows them to be found numerically.

In the works of Zubov and his scientific group (see [6]), a complete analysis of free motions of a gyrostat and motions of a gyrostat with a constant external torque was provided. A classification of types of gyrostat motions was given, and the domains of values of constructive parameters and domains of initial conditions are divided into subdomains corresponding to the motions of only one type.

In addition, V. I. Zubov has developed special approaches to constructing conservative numerical methods for integration of equations of motion of a rigid body [6, 7]. These approaches are based on the introduction of controls in the computational process to provide preservation of qualitative characteristics (integrals, integral invariants, stability, etc.) when passing from the differential equations to the corresponding difference ones.

2.2 Attitude control of a rigid body

V. I. Zubov has considered the problem of active attitude control of a rigid body in the general nonlinear statement [6, 8–16]. He has proposed new approaches to the synthesis of control torques providing stabilization of prescribed orientations of a body. Moreover, he has fulfilled complete investigation of qualitative behavior of solutions for the corresponding closed-loop systems.

In particular, consider Zubov's approach to the problem of monoaxial stabilization of a body (see [16]). Let a rigid body rotating around its mass center O with angular velocity $\boldsymbol{\omega}$ be given. Denote by $Oxyz$ the principal central axes of inertia of the body. The attitude motion of the body under a control torque $\mathbf{M}$ is described by the Euler equations

$$\boldsymbol{\Theta}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \boldsymbol{\Theta}\boldsymbol{\omega} = \mathbf{M}. \tag{5}$$

Here $\boldsymbol{\Theta}$ is the inertia tensor of the body on the axes $Oxyz$.

Let the unit vectors $\mathbf{s}$ and $\mathbf{r}$ be given, and the vector $\mathbf{s}$ be constant in the inertial space and the vector $\mathbf{r}$ be constant in the body-fixed frame. Then the vector $\mathbf{s}$ rotates with respect to the coordinate system $Oxyz$ with the angular velocity $-\boldsymbol{\omega}$. Hence,

$$\dot{\mathbf{s}} = -\boldsymbol{\omega} \times \mathbf{s}. \tag{6}$$

Thus, we consider the differential system consisting of the Euler dynamic equations (5) and the Poisson kinematic equations (6). It is required to design a control torque $\mathbf{M}$ providing monoaxial stabilization of the body: the corresponding closed-loop system should admit the asymptotically stable equilibrium position

$$\boldsymbol{\omega} = \mathbf{0}, \qquad \mathbf{s} = \mathbf{r}. \tag{7}$$

V. I. Zubov has proposed to choose a control torque in the form

$$\mathbf{M} = -\boldsymbol{\omega} + k\mathbf{r} \times \mathbf{s}. \tag{8}$$

Here k is a positive constant. It should be noted that, for such a control, the system (5), (6), along with (7), has the equilibrium position

$$\boldsymbol{\omega} = \mathbf{0}, \qquad \mathbf{s} = -\mathbf{r}. \tag{9}$$

With the aid of the Lyapunov function

$$V = \left(\boldsymbol{\omega}^\top \boldsymbol{\Theta}\boldsymbol{\omega} + k\|\mathbf{s} - \mathbf{r}\|^2\right)/2 \tag{10}$$

the following theorem was proved (see [16]).

Theorem 2.2 *Let a control torque be defined by the formula* (8)*. Then the equilibrium position* (7) *is asymptotically stable, whereas the equilibrium position* (9) *is unstable. In addition, any motion of the closed-loop system different from the equilibrium position* (9)*, for an appropriate choice of the coefficient* k*, possesses the property* $\boldsymbol{\omega} \to \mathbf{0}$*,* $\mathbf{r} \to \mathbf{s}$ *as* $t \to +\infty$*.*

V. I. Zubov has considered also the problem of scanning a body axis in accordance with the prespecified program [13, 16]. It was assumed that a unit vector $\mathbf{s}_0(t)$ rotates in the inertial space with a given angular velocity $\boldsymbol{\omega}_{\mathbf{0}}(t)$. A control torque should provide the fulfilment of the conditions $\boldsymbol{\omega}(t) \to \boldsymbol{\omega}_{\mathbf{0}}(t)$, $\mathbf{r}(t) \to \mathbf{s}_0(t)$ as $t \to +\infty$.

It was proved (see [13, 16]) that the required control can be chosen in the form

$$\mathbf{M} = \boldsymbol{\omega}_0(t) - \boldsymbol{\omega} + \boldsymbol{\Theta}\dot{\boldsymbol{\omega}}_0(t) + \boldsymbol{\omega}_0(t) \times \boldsymbol{\Theta}\boldsymbol{\omega} + k\mathbf{r} \times \mathbf{s}_0(t), \quad k = \text{const} > 0.$$

Similar approaches were developed for the problem of triaxial stabilization of a rigid body [8, 11, 13, 14].

To construct the control laws ensuring scanning body axes in accordance with the prespecified program, it is necessary to detect first the required elements of body motions. In [3, 6, 13], the problems of determination of orientation of a satellite and localization of motions were solved. New approaches to detect orientation of a satellite via two known physical vectors (for example, direction to the Earth or to the Sun, magnetic field vector, etc.) were proposed.

In addition, V. I. Zubov has developed new methods for the attitude control of a rigid body with the aid of flywheels and rotors connected with the body [6, 11–13, 16]. These methods are based on finding the motions of the carried bodies which create the Coriolis force moments and the moments of relative forces of inertia providing the prescribed motions of the carrying body. For a set of such problems, stationary motions were determined, and stability of these motions was investigated.

Furthermore, for the bodies with liquid-filled cavities and for the bodies with flexible constructions, original mathematical models based on the ordinary differential equations were suggested. For such models, the analytical constructions of controls providing given rotational motions of the carrier were obtained (see [6]).

2.3 Applications

Zubov's investigations were always aimed at applications. He has efficiently exploited the developed methods for the solution of the following practical problems:

(i) the design of precision control systems of spacecraft positions for the "Proton" system;

(ii) the design of control systems for the rotational motion of spacecrafts for the precision orientation of sensitive axes of devices on the base of magneto-hydrodynamic control systems with the use of conducting fluids in feedback contours.

3 Some Extensions of Zubov's Results

The present paper does not claim to provide a comprehensive review of all the numerous publications exploring and developing Zubov's ideas and results. Here we confine ourselves to just a few developments of Zubov's heritage.

3.1 Construction of strict Lyapunov functions

It is worth mentioning that Zubov's results on attitude control of a rigid body are based on the constructing weak Lyapunov functions. Derivatives of these functions with respect to the systems under study are only nonnegative. It is known, that such Lyapunov functions are not well suited to the robustness analysis since their negative semidefinite derivatives along the trajectories could become positive under arbitrarily small perturbations of the dynamics. This has motivated the development of the methods for constructing strict Lyapunov functions, i.e., the functions with negative definite derivatives.

E. Ya. Smirnov has proposed an approach for transforming the weak Lyapunov functions constructed by Zubov into the strict ones (see [17]).

For instance, for the problem of monoaxial stabilization of a rigid body, he has suggested the following modification of the Lyapunov function (10):

$$\widetilde{V} = \left(\boldsymbol{\omega}^\top \boldsymbol{\Theta}\boldsymbol{\omega} + k\|\mathbf{s} - \mathbf{r}\|^2\right)/2 - \gamma\boldsymbol{\omega}^\top \boldsymbol{\Theta}\mathbf{P}(\mathbf{r} \times \mathbf{s}), \tag{11}$$

where γ is a positive parameter, $\mathbf{P}$ is a constant matrix [17]. It was proved that, if the matrix $\mathbf{P}^\top + \mathbf{P}$ is positive definite, then, for sufficiently small values of γ, the derivative of (11) along the solutions of the system (5), (6) closed by the control (8) is negative definite with respect to the variables $\boldsymbol{\omega}$ and $\mathbf{r} - \mathbf{s}$.

On the basis of Smirnov's approach, in his works and the works of his scientific group (see [17–20]), various extensions of Zubov's results were obtained. In particular, the methods of robust attitude control were developed for the cases where the inertia tensor of a body and the torques acting on the body are given with some errors [18].

Moreover, in [21–24], strict Lyapunov functions were constructed in the problems of monoaxial and triaxial stabilization of rigid bodies with essentially nonlinear control torques.

3.2 Stabilization with respect to a part of variables

It is known [25, 26], that the perturbations resulting in the attitude deviations of a rigid body from a given position can be treated as the uncontrolled variables when solving the problem of partial stabilization of stationary motions of the body via the flywheels (rotors). It was shown (see [26]) that a flywheel may also be used for the partial stabilization of the permanent rotation of a solid.

Another interesting partial control problem is the problem of "passage" of a solid through a given angular position in the three-dimensional inertial space. This problem is encountered, for example, in a quickly reorienting spacecraft for implementing instant actions (photographing, firing, data transmission, etc.) when the body reaches the desired angular position. For an asymmetric solid, this problem was solved in [27].

Nonlinear game problem of monoaxial reorientation for an asymmetric rigid body with the internal torques applied to the flywheels connected with the body was considered in [28]. The estimates for admissible levels of noise depending on the control constraints were found based on the method proposed in [29], where the control is carried out via the moments of external forces realized by the engines.

Some sufficient conditions of the partial stability and partial asymptotic stability of programmed motions of a rigid body were derived in [22, 23] with the aid of the comparison method.

3.3 Nonstationary moments of inertia and control torques

In the papers [21, 30], the problems of the monoaxial and triaxial stabilization of a rigid body with time-varying moments of inertia were considered, and sufficient conditions of the asymptotic stability of prescribed orientations are found. It is worth noting that, in [30], weak Lyapunov functions and the method of limiting equations were used, whereas the results of [21] are based on the application of special constructions of strict Lyapunov functions and the theory of differential inequalities.

Furthermore, in [21–23, 31], some problems of attitude stabilization of a rigid body with the use of restoring and dissipative torques were studied for the case where control torques evolve with time. In particular, the possibility of implementing the control systems in which the restoring or dissipative torques tend to zero as time increases was investigated. It is well known that such an evolution could result in new dynamic effects and difficulties of substantiation. Both cases of linear and essentially nonlinear controls were considered. With the aid of the Lyapunov direct method and the comparison method, the conditions were derived under which we can guarantee stability of given equilibrium positions of a body despite the evolution of control torques.

3.4 Application of the averaging technique

In [24, 32], some problems of attitude dynamics of a rigid body influenced by the linear dissipative torque, homogeneous (linear or nonlinear) restoring torque and nonstationary perturbations with zero mean values were studied. It was assumed that the orders of homogeneity of perturbations coincide with those of the components of the restoring torque. The averaging technique was applied and developed for the problems. Original constructions of nonstationary Lyapunov functions taking into account the structure of perturbations acting on the body were proposed. With the aid of these functions, sufficient conditions for the asymptotic stability of the body equilibrium positions were derived. It was proved that, in the case of linear restoring and perturbing torques, the destabilizing effect of nonstationary perturbations can be compensated via introducing a sufficiently large multiplier at the vector of dissipative torque, whereas, for the nonlinear case, to guarantee the asymptotic stability of the equilibrium positions, it is not necessary to use such a parameter.

3.5 Control problems with incomplete feedback

Consider the problem of synthesis of the controls ensuring the asymptotic stability of rotation motion of a body around one of the principal central axes of inertia with a given angular velocity $\boldsymbol{\omega} = \boldsymbol{\omega}_0$. In the presence of complete feedback, when there are three angular velocity sensors measuring its projections onto the selected axes, this problem was solved in [8]. At the same time, A.M. Letov posed the problem of constructing such a control using a smaller number of sensors [33]. Letov's problem, continuing and developing the results of V.I. Zubov, was treated in a number of works. Some of them are mentioned below:

(1) For $\boldsymbol{\omega}_0 = \mathbf{0}$, this problem was solved in [34], and for the case of monoaxial stabilization, in [35].

(2) For $\boldsymbol{\omega}_0 \neq \mathbf{0}$, the monoaxial stabilization problem was solved in a local setting in [36] and [37] for the case where the vector $\boldsymbol{\omega}_0$ is directed along the main axis of inertia of a body.

(3) In the case of two sensors, the problem is solved in [38] for almost all inertia tensors.

(4) In the case of one sensor, the problem is solved in [39] in the linear approximation using a dynamical controller.

(5) The case of one sensor was investigated nonlocally in [40] taking into account nonlinear terms. The investigation is based on the Lyapunov functions method.

(6) In [41], the case of monoaxial stabilization was investigated. It was suggested to choose the required controls as linear functions with respect to the deviations of the projections of the angular velocity vectors from the prescribed values. Conditions were obtained under which such a control provides the asymptotic stability of rotation around the large, small or medium axis of inertia.

3.6 Equilibria of a gyrostat satellite

Permanent Zubov's interest in gyroscopic systems was inspired by their applications in shipbuilding, air- and spacecraft industry [4]. One of the gyroscopic objects is a gyrostat-satellite, the analysis of the dynamics of which attracted the attention of V. I. Zubov and his followers.

Bifurcations of the relative equilibria of a gyrostat satellite moving in a circular Keplerian orbit was investigated in [42] for a special case of the alignment of its gyrostatic moment. The whole set of equilibria with respect to the orbital system of coordinates of the gyrostat satellite was determined using the given moments of inertia, the value of the gyroscopic moment and the direction cosines of the axis of rotation of the flywheel and the changes in this set are investigated as a function of the bifurcation parameter, i.e., the magnitude of the gyrostatic moment of the system. A parametric analysis of the relative equilibria of the three possible classes of equilibria for a system in a circular orbit in a central Newtonian force field is carried out using computer algebra facilities.

The usage of LinModel software package and the symbolic-numerical modeling functions of the Mathematica Computer Algebra System has also proved to be fruitful in the stability investigations of the orbital gyrostat equilibria [43]. By means of Lyapunov's approach, the regions in the space of input parameters are determined, where the stability, instability, or gyroscopic stabilization of relative equilibria of a prolate axisymmetric orbital gyrostat with a constant gyrostatic moment vector are ensured.

A new geometric approach to the analysis of the set of relative gyrostat equilibria is developed in [44]. It is proposed to determine the relative gyrostat equilibria in the corresponding three-dimensional Euclidean space using special aggregated parameters of the system by the coordinates of the intersection points of two pairs of corresponding hyperbolic cylinders with the sphere of unit radius. It is shown that, for the arbitrary values of the gyrostatic moment and other parameters of the system, there are at least eight different relative equilibria.

3.7 A gyrostat satellite in the gravitational and magnetic fields

The attitude motion of a gyrostat satellite is considered in [45] taking into account its interaction with the Earth's magnetic field through its own magnetic moment. The existence of a relative equilibrium of the gyrostat satellite in a special coordinate system associated with the geomagnetic induction vector is proved. The implementation of one particular case of such motion is given. Based on the numerical integration of the differential equations of the perturbed motion, the obtained stability conditions for the gyrostat satellite are analyzed.

A gyrostat satellite moving in a circular Keplerian orbit in the plane of geomagnetic equator is considered in [46]. The gyrostat is equipped with a flywheel, has an electrostatic charge and its own magnetic moment. The attitude motion of the gyrostat under the action of the Lorentz and magnetic torques is studied. It is shown that in the case of dynamic and electromagnetic symmetry of the gyrostat, the problem reduces to quadratures by constructing four first integrals. The motion of the gyrostat axis of symmetry is studied, and its geometric interpretation is given. The same gyrostat in a weakly elliptic orbit is considered in [47]. The reversibility of the differential system with three fixed sets is established. The properties of the symmetric periodic oscillations are analyzed. It is proved that during the transition from a circular orbit to a weakly elliptical one, a bifurcation of the family of symmetric oscillations of the circular problem occurs and two isolated symmetric oscillations are generated.

A gyrostat satellite with a triaxial inertia ellipsoid in a weakly elliptic orbit with small inclination is considered in [48]. The attitude motion of the gyrostat under the influence of the Lorentz, magnetic and gravitational torques is studied. It is shown that the problem is reversible with two fixed sets. In the case of an isoinertial gyrostat, a third fixed set appears. Two of these three sets correspond to the sets of the degenerate problem [46]. It was found out which sets of symmetric oscillations of the degenerate problem bifurcate and generate isolated oscillations.

3.8 Attitude control of a rigid body using powered gyroscopes

In addition, the development of Zubov's ideas is contained in a series of works by E.Ya. Smirnov and his scientific group devoted to the attitude control of a rigid body using powered gyroscopes [17,18]. For instance, in [18], the problem of triaxial attitude control of a rigid body (carrier) using three pairs of two-degree-of-freedom powered gyroscopes is considered. The errors are taken into account in the construction of the carrier and gyroscopes, as well as the errors in the installation of gyroscopes relative to the carrier. The controls are found that solve the problem of the triaxial orientation of a solid.

3.9 An extension of the classes of stabilizing controls

When solving spacecraft control problems, in numerous cases it is necessary to take into account such effects as the discrete nature of the receipt of information about the state of control objects and its transfer to control devices, the specifics of the functioning of executive devices and the delay in feedback laws. This results in an extension of the classes of applied controls. In particular, in [17, 18], the discrete-time and relay-type control torques providing monoaxial and triaxial stabilization of a rigid body were proposed; in [18, 49], the impulse controls were applied; some problems of the attitude control for the case of delay in feedback laws were solved in [26, 49, 50].

3.10 Development of conservative numerical methods

Among the developments of Zubov's results on the theory of conservative methods for the numerical integration of differential systems, it is worth mentioning the method of numerical continuation with respect to parameters for constructing periodic motions. For an autonomous Hamiltonian system, this method is described in [51]. It has wide applications to the problems of rigid body attitude dynamics in which the numerical construction of the families of periodic motions generated from the regular precessions of a dynamically symmetric satellite is of practical interest. A modification of this method was proposed in [52]. This modification allowed to significantly increase the speed of calculations as well as the accuracy of numerical calculations.

4 Conclusion

Vladimir Zubov was a prominent scholar, engineer and university lecturer. In the previous sections we have reviewed just only one area of scientific activity of him and his successors.

Zubov is the author of about 200 publications including 31 monographs and text books. He was an advisor for 20 DSc and about 100 PhD dissertations. Under Zubov's supervision, a worldwide famous school in control theory was developed in St. Petersburg.

In 1968, V. I. Zubov became the USSR State Prize winner for his pioneer works in control theory. In 1981, he was elected a corresponding member of the Soviet Union Academy of Sciences, and in 1998, he was awarded the title of the Honored Scholar of the Russian Federation. In 1996, the Zubov scientific school "Processes of control and stability" was the winner of the competition for the State support of leading scientific schools of Russia. In 2001, the Research Institute of Computational Mathematics and Control Processes of St. Petersburg State University was named after him.

For outstanding merits to the world science, Zubov's name was perpetuated as the name of the minor planet 'ZUBOV 10022'. This asteroid has a size of 6 km, a brightness of 13.8 magnitude, and the greatest orbit's semiaxis of 2.369 astronomical units.

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