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Oscillation Criteria for Delay Equations with Several Non-Monotone Arguments

I. P. Stavroulakis [*,1,2], Zh. Kh. Zhunussova [2], L. Kh. Zhunussova [3] and K. Dosmagulova [2]

[1] *Department of Mathematics, University of Ioannina, 451 10 Ioannina, HELLAS (Greece)*
[2] *Al-Farabi Kazakh National University, Almaty, 050040 Kazakhstan*
[3] *Abai Kazakh National Pedagogical University, Almaty, 050010, Kazakhstan*

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Abstract: Consider the first-order linear differential equation with several retarded arguments $x'(t) + \sum_{i=1}^{m} p_i(t)\, x(\tau_i(t)) = 0,\ t \ge t_0$, where the functions $p_i,\ \tau_i \in C\left([t_0, \infty), \mathbb{R}^+\right)$, for every $i = 1, 2, \ldots, m,\ \tau_i(t) \le t$ for $t \ge t_0$ and $\lim_{t\to\infty} \tau_i(t) = \infty$. In this paper we review the most interesting sufficient conditions under which all solutions oscillate. An example illustrating the results is given.

Keywords: *oscillation; retarded; differential equations; non-monotone arguments.*

Mathematics Subject Classification (2010): Primary 34K11, Secondary 34K06.

1 Introduction

Consider the first-order linear differential equation with several non-monotone retarded arguments

$$x'(t) + \sum_{i=1}^{m} p_i(t)\, x(\tau_i(t)) = 0,\ t \ge t_0, \tag{1.1}$$

where the functions $p_i,\ \tau_i \in C([t_0, \infty), \mathbb{R}^+)$, for every $i = 1, 2, \ldots, m$, (here $\mathbb{R}^+ = [0, \infty)$), $\tau_i(t) \le t$ for $t \ge t_0$ and $\lim_{t\to\infty} \tau_i(t) = \infty$.

Let $T_0 \in [t_0, +\infty)$, $\tau(t) = \min\{\tau_i(t) : i = 1, \ldots, m\}$ and $\tau_{-1}(t) = \sup\{s : \tau(s) \le t\}$. By a solution of the equation (1.1) we understand a function $x \in C([T_0, +\infty), \mathbb{R})$, continuously differentiable on $[\tau_{-1}(T_0), +\infty]$ and that satisfies (1.1) for $t \ge \tau_{-1}(T_0)$. Such a solution is called *oscillatory* if it has arbitrarily large zeros, and otherwise it is called *non-oscillatory.*

For the general theory the reader is referred to [9, 11, 12, 17].

The oscillatory behavior of functional differential equations has been the subject of many investigations. See, for example, [1–20] and the references cited therein.

* Corresponding author: mailto:ipstav@uoi.gr