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Estimates of Accuracy for Asymptotic Soliton-Like Solutions to the Singularly Perturbed Benjamin-Bona-Mahony Equation

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Abstract: The paper deals with the singularly perturbed Benjamin-Bona-Mahony equation with variable coefficients. It plays an important role in various applications, in particular, for the description of waves in liquid. The equation appears in mathematical modeling of the wave processes in the media with small dispersion and variable characteristics. In the case of constant coefficients, this equation is known as the regularized long-wave equation or the regularized Korteweg-de Vries equation. We study the problem of estimating the difference between the exact solution and asymptotic soliton-like solution to the Cauchy problem for the singularly perturbed Benjamin-Bona-Mahony equation with variable coefficients. The initial data for the Cauchy problem are defined according to the concept of asymptotic soliton-like solution. It means that the approximate solutions are deformations of the soliton solutions to the Benjamin-Bona-Mahony equation with corresponding constant coefficients.
Asymptotic estimates for the difference between the exact solution to the Benjamin-Bona-Mahony equation and the N-th approximation for the asymptotic soliton-like solution are obtained. In particular, the case of the main term of the solution is considered in detail. Similarly to the case of the singularly perturbed Korteweg-de Vries equation with variable coefficients these estimates are local. Nevertheless, they show that the asymptotic soliton-like solutions constructed through the nonlinear WKB method for the singularly perturbed Benjamin-Bona-Mahony equation with variable coefficients are sufficiently suitable as approximate solutions.

Keywords: *Benjamin-Bona-Mahony equation; asymptotic solutions; soliton-like solutions; Cauchy problem; asymptotic estimates.*

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