Periodic Solutions in Non-Homogeneous Hill Equation

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Abstract: Properties of T and $2T$ periodic solutions in the homogeneous Hill equation have been entirely determined, but there is hardly any information about the existence of periodic solutions with different period. In this work, kT periodic solutions in the Hill equation will be explicitly characterized, here k is a natural number. Moreover, it will be shown that those kT periodic solutions become unstable when the system is forced with a function having the same period (or an integer multiple of it) of any of those solutions. As a consequence, two types of instability will be presented for the first time on the Ince-Strutt diagram: the well-known parametric resonance and the linear resonance due to the forcing signal.

Keywords: *non-homogeneous Hill equation; kT-periodic solutions; linear and parametric resonance; Ince-Strutt diagram; Floquet multipliers.*

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1 Introduction

1.1 Hill equation

The general class of homogeneous second order linear differential equations with real periodic coefficients can be characterized by the Hill equation (1), it describes dynamical systems with intrinsic periodicity and parametric behaviour such as the modulation of radio carrier waves, transverse vibrations of a tense elastic member, the stability of a periodic motion in a non-linear system (linearization in a neighbourhood of a periodic motion) and the focus and defocus of particle beams in particle accelerators. Also, this equation can be seen as a particular case of the Schrödinger equation with periodic potential.

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