Hausdorff Dimension Estimates by Use of a Tubular Carathéodory Structure and Their Application to Stability Theory

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Abstract: The paper is concerned with upper bounds for the Hausdorff dimension of flow invariant compact sets on Riemannian manifolds and the application of such bounds to global stability investigations of equilibrium points. The proof of the main theorem uses a special Carathéodory dimension structure in order to get contraction conditions for the considered Carathéodory measures which majorize the Hausdorff measures. The Hausdorff dimension bounds in the general case are formulated in terms of the eigenvalues of the symmetric part of the operator which generates the associated system in normal variations with respect to the direction of the vector field. For sets with an equivariant tangent bundle splitting dimension bounds are derived in terms of uniform Lyapunov exponents. A generalization of the well-known theorems of Hartman-Olech and Borg is given.

Keywords: *Hausdorff dimension; Carathéodory dimension structure; outer measures via tube covers; system in normal variations; global stability; uniform Lyapunov exponents; equivariant tangent bundle splitting; Riemannian manifolds.*

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