Periodic Solutions of a Singular Lagrangian System Related to Dispersion-Managed Fiber Communication Devices

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Abstract: We prove the existence of periodic solutions to a certain singular Lagrangian system that describes the evolution of the optical pulse width and chirp for so-called dispersion-managed solitons.

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1 Introduction and Main Results

In data communication systems like transoceanic transmission along a fiber cable, there is increasing demand to achieve transmission rates as high as possible, mainly to the extensive use of the internet. To do so, a recent approach is to utilize non-linear light-wave communications with suitable periodic amplifications to compensate for loss and dispersive effects. The transmission of such optical signal is described by

\[ i\Psi_z - \frac{1}{2} \beta_2(z)\Psi_{tt} + \sigma(z)|\Psi|^2\Psi = iG(z)\Psi, \]  

(1)

see [6, 8, 9]. Here \( \Psi = \Psi(z,t) \) is some complex-valued envelope function of the original electric field, \( t \) is time, and \( z \) is the longitudinal coordinate of the fiber cable, which should be thought of to be a periodic variable, since both amplification and dispersion repeat periodically. Moreover, \( G(z) \) accounts for both loss and amplification in the fiber, whereas \( \beta_2(z) \) is related to the dispersion; \( \sigma(z) \) is some additional function.

The transformation \( \Psi(z,t) = A(z,t) \exp \left( \int G(z') \, dz' \right) \) removes the term on the right-hand side of (1) to yield the nonlinear Schrödinger equation

\[ iA_z + d(z)A_{tt} + c(z)|A|^2A = 0, \]  

(2)

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