Stability and Hopf Bifurcation in Differential Equations with One Delay

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Abstract: A class of parameter dependent differential equations with one delay is considered. A decomposition of the parameter space into domains where the corresponding characteristic equation has a constant number of zeros with positive real part is provided. The local stability analysis of the zero solution and the computation of all Hopf bifurcation points with respect to the delay is given.

Keywords: Nonlinear delay differential equations; zeros of quasi-polynomials; local stability; Hopf bifurcation

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1 Introduction

Local stability and bifurcation analysis of systems of nonlinear differential equations with one time delay of the following type

\[
\dot{x}(t) = Ax(t) + Bx(t - \tau) + F(x(t), x(t - \tau)),
\]

where \(\tau \geq 0\); \(A, B \in \mathbb{R}^{n \times n}\), \(F \in C^k(\mathbb{R}^n \times \mathbb{R}^n, \mathbb{R}^n)\), \(k \geq 1\), \(F(0, 0) = DF(0, 0) = 0\), often leads to the consideration of quasi-polynomials \(\Phi_{\tau,\lambda}: \mathbb{C} \to \mathbb{C}\); \(\tau \geq 0\), \(\lambda \in \mathbb{C}\), given by

\[
\Phi_{\tau,\lambda}(s) := (s + 1) \exp(\tau s) - \lambda.
\]

In this context it is of particular relevance to know how the zeros of \(\Phi_{\tau,\lambda}\) are distributed in the complex plane, whether they lie in the left or right half plane, and finally, how they depend on the parameters \(\tau\) and \(\lambda\).

The objective of this work is to divide the \(\tau\)-halfline and the \(\lambda\)-plane into domains where \(\Phi_{\tau,\lambda}\) has a constant number of zeros with positive real part and to investigate the