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On the Design of Nonlinear Controllers for Euler-Lagrange Systems

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Abstract: The dynamics are studied of nonlinear feedback loops for the set point control of Euler-Lagrange (EL) systems. A class of controllers is considered that possess a linear dynamic component and several nonlinear amplifiers. Frequency domain conditions are presented for nonoscillatory behaviour of the closed loop, by which is meant that for increasing time all bounded solutions converge to one of the system's equilibrium states. The results constitute a systems theoretical basis for a new controller design method for EL systems.

Keywords: *Euler-Lagrange systems; nonlinear control; Liapunov's method; convergence criteria; stability regions.*

Mathematics Subject Classification (2000): 34D20, 70H35, 70K15, 70Q05.

1 Introduction

Euler-Lagrange systems constitute the outcome of a powerful mathematical modelling technique for dynamic processes, the variational method [4]. Their structural properties and constraints have been exploited to develop practically meaningful controller design procedures including Liapunov based methods [9], passivity based control [6] or the stabilization scheme of backstepping [1]. Most of the literature dealing with set point regulation of EL systems concentrates on the global asymptotic stabilization of a unique closed loop equilibrium state, elaborating on such fundamental concepts as potential energy shaping and damping injection [8]. Nevertheless there remain several drawbacks that stymie the utilization of these methodologies in practical applications. For example, the global stabilization of a unique equilibrium point often requires control inputs beyond the physical saturation constraints of the actuators. This has led to the development of saturated controllers which apply to EL systems with limited growth rates of the potential energy functions for large position values [3].

In this paper we consider a class of nonlinear feedback controllers for EL systems that allow the existence of several closed loop equilibria. Sufficient conditions are established for nonoscillatory behaviour of the loop, by which is meant that for increasing time every bounded solution converges to one of the equilibria. If for increasing time all solutions remain bounded, nonoscillatory behaviour implies the convergence of all solutions to the set of equilibria, i.e. the set of equilibria is globally convergent. The conditions for nonoscillatory behaviour are much less restrictive than those for the global asymptotic stabilization of a unique equilibrium point. Thus they constitute a systems theoretical basis for an alternative controller design method in cases where a closed loop phase portrait possessing several equilibrium states is acceptable or desirable.

Following some background concepts in Section 2, Section 3 presents the basic conditions which guarantee closed loop nonoscillatory behaviour and global convergence of the set of equilibria. Sections 4 and 5 contain their application to a dynamic output feedback controller possessing several nonlinear amplifiers, some comments regarding controller design and some special cases. Section 6 discusses an application to EL controllers. In Section 7 a simple example is worked to illustrate the proposed concepts. The paper terminates with an overview of possible extensions to the theory and of further work under development.

2 EL Systems: Basic Concepts and Assumptions

Consider an EL system [4]

$$\frac{\partial \mathcal{L}}{\partial q}(q,\dot q) - \frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot q}(q,\dot q)\right] - \frac{\partial \mathcal{F}}{\partial \dot q}(\dot q) + Mu = 0, \tag{1}$$

where $q \in R^m$ is a vector of generalized coordinates, $\mathcal{L}(q,\dot q) \triangleq \mathcal{T}(q,\dot q) - \mathcal{V}(q)$ is the Lagrangian and $\mathcal{F}(\dot q)$ is Rayleigh's dissipation function. $u \in R^r$ denotes a linearly entering input force. We assume that the kinetic energy $\mathcal{T}(q,\dot q)$ and the potential energy $\mathcal{V}(q)$ belong to the class of C^1-functions (continuous with continuous partial derivatives w.r.t. their arguments), that rank $M = r \le m$ and that the Lipschitz conditions are satisfied which ensure the existence and the uniqueness of the solutions of (1) for given initial conditions and for a given input $u(.)$. The system is called fully actuated if $r = m$, otherwise it is underactuated. Following Meirovitch [4] we assume that $\mathcal{T}(q,\dot q)$ depends quadratically on the components of $\dot q$:

$$\mathcal{T}(q,\dot q) = \frac{1}{2}\,\dot q' D(q)\dot q + b'(q)\dot q + c(q), \tag{2}$$

where the generalized inertia matrix $D(q) = D'(q) \in R^{m\times m}$ is positive definite, $b(q) \in R^m$ and $c(q) \in R$. Defining the Hamiltonian as

$$\mathcal{H}(q,\dot q) \triangleq \frac{1}{2}\,\dot q' D(q)\dot q + \mathcal{V}(q) - c(q) \tag{3}$$

it is easily verified that along the solutions of (1):

$$\frac{d\mathcal{H}}{dt}(q,\dot q) = -\dot q'\,\frac{\partial \mathcal{F}}{\partial \dot q}(\dot q) + \dot q' Mu. \tag{4}$$

We assume that

$$\dot q' \frac{\partial \mathcal{F}}{\partial \dot q}(\dot q) \geq 0; \quad \forall\, \dot q \in R^m. \tag{5}$$

If

$$\dot q' \frac{\partial \mathcal{F}}{\partial \dot q}(\dot q) > 0; \quad \forall\, \dot q \neq 0 \tag{6}$$

then the EL system is said to be fully damped. Otherwise it is underdamped. Observing that the system state is $x \triangleq \begin{bmatrix} q \\ \dot q \end{bmatrix} \in R^{2m}$, (4) shows that (1) is a dissipative system [11] with storage function $\mathcal{H}(q, \dot q)$ and supply $\dot q' M u$. We shall assume that the output

$$w \triangleq M' q \in R^r \tag{7}$$

is available for feedback. (1), (7) define an EL system with collocated actuator-sensor control [7].

3 Closed Loop Nonoscillatory Behaviour and Global Convergence

Let

$$\dot z = \varphi(z, w), \tag{8}$$
$$u = \psi(z, w) \tag{9}$$

with state $z \in R^n$ be a feedback controller for (1), (7). Let $x_c \triangleq \begin{bmatrix} x \\ z \end{bmatrix} \in R^{2m+n}$ be the closed loop state vector. Suppose a scalar function $V(z, w) \in C^1$ can be found such that along the solutions of (8)

$$\dot z' \frac{\partial V}{\partial z}(z, w) = \varphi'(z, w) \frac{\partial V}{\partial z}(z, w) \leq 0; \quad \forall\, z \in R^n, \quad \forall\, w \in R^r. \tag{10}$$

Define

$$V_c(x_c) \triangleq \mathcal{H}(q, \dot q) + V(z, w). \tag{11}$$

It follows that

$$\begin{aligned} \dot V_c(x_c) &= -\dot q' \frac{\partial \mathcal{F}}{\partial \dot q}(\dot q) + \dot q' M u + \dot z' \frac{\partial V}{\partial z}(z, w) + \dot w' \frac{\partial V}{\partial w}(z, w) \\ &= -\dot q' \frac{\partial \mathcal{F}}{\partial \dot q}(\dot q) + \dot z' \frac{\partial V}{\partial z}(z, w) \leq 0; \quad \forall\, x_c \in R^{2m+n}, \end{aligned} \tag{12}$$

if we choose

$$u = \psi(z, w) \triangleq -\frac{\partial V}{\partial w}(z, w). \tag{13}$$

By (12), $V_c(x_c)$ is a global Liapunov function for the closed loop system. Invoking Lasalle's invariance principle [2] it follows that every solution $x_c(t)$ that remains bounded for $t \geq 0$ will for $t \to +\infty$ converge to the largest invariant subset $\mathcal{M}$ of the closed loop state space where

$$-\dot{q}' \frac{\partial \mathcal{F}}{\partial \dot{q}}(\dot{q}) + \dot{z}' \frac{\partial V}{\partial z}(z, w) \equiv 0. \tag{14}$$

Suppose we can select $V(z, w)$ such that $\mathcal{M}$ consists of the set of the closed loop equilibria. Then every solution $x_c(t)$ that remains bounded $t \geq 0$ will converge to an equilibrium point. A system possessing this property will be called nonoscillatory. Every solution of a nonoscillatory system either tends to infinity or converges to an equilibrium point as $t \to +\infty$. It cannot perform complicated motions such as periodic oscillations or chaos. As a corollary to the above we have

Lemma 3.1 *If the EL system (1) is fully damped and if in addition to (10),*

$$\dot{z}' \frac{\partial V}{\partial z}(z, w) = 0 \quad \Longleftrightarrow \quad \dot{z} = 0 \tag{15}$$

then the closed loop (1), (7), (8), (13) is nonoscillatory.

If all solutions of a nonoscillatory system remain bounded for $t \geq 0$, then every solution converges to an equilibrium state as $t \to +\infty$. In other words the set of the equilibria is globally convergent. The boundedness of solutions can often easily be proved, for example using a suitable Liapunov function. Specifically we have

Lemma 3.2 *If in addition to the conditions of Lemma 3.1, $V_c(x_c)$ is radially unbounded then the set of the closed loop equilibria is globally convergent.*

4 Controllers with Several Arbitrary Nonlinear Amplifiers

Consider a controller with state dynamics of the form

$$\dot{z} = Az - Bf(\sigma) + \eta(w), \tag{16}$$

$$\sigma = C'z + \zeta(w), \tag{17}$$

where $A \in R^{n\times n}$ is nonsingular; $B,\ C \in R^{n\times s}$; $\eta \in R^n$; $\zeta \in R^s$; $f(\sigma) = \operatorname{col}[f_i(\sigma_i);\ i = 1, \dots, s]$. Let

$$V(z, w) \triangleq z'Pz + \int_0^{\sigma} f'(\theta)\bar{\alpha}\, d\theta + z'p(w) + \mu(w)$$

with $P = P' \in R^{n\times n}$; $\bar{\alpha} = \operatorname{diag}(\alpha_i) \in R^{s\times s}$; $\theta \in R^s$; $p(w) \in R^n$ and $\mu(w) \in R$. Partial differentiation of $V(z, w)$ along the solutions of (16), (17) produces

$$\begin{aligned}
\left(\frac{\partial V}{\partial z}\right)' \dot{z} &= \dot{z}'Pz + z'P\dot{z} + f'(\sigma)\bar{\alpha}C'\dot{z} + \dot{z}'p(w) \\
&= \dot{z}'PA^{-1}[\dot{z} + Bf(\sigma) - \eta(w)] \\
&\quad + [\dot{z} + Bf(\sigma) - \eta(w)]'A^{-1\prime}P\dot{z} + f'(\sigma)\bar{\alpha}C'\dot{z} + \dot{z}'p(w) \\
&= \dot{z}'[PA^{-1} + A^{-1\prime}P]\dot{z} + \dot{z}'[2PA^{-1}B + C\bar{\alpha}]f(\sigma) \\
&\quad + \dot{z}'[p(w) - 2PA^{-1}\eta(w)].
\end{aligned}$$

Defining $W \triangleq A^{-1\prime} P A^{-1}$ and choosing

$$A'W + WA = -QQ' - \varepsilon I; \quad \varepsilon > 0, \tag{18}$$

$$2WB + A^{-1\prime} C\bar{\alpha} = 0, \tag{19}$$

$$p(w) = 2A'W\eta(w) \tag{20}$$

results in

$$\left(\frac{\partial V}{\partial z}\right)' \dot{z} = -\dot{z}'QQ'\dot{z} - \varepsilon \dot{z}'\dot{z} \tag{21}$$

which satisfies (10) and (15). By virtue of the Kalman-Yacubovich-Popov main lemma [10] the system (18), (19) has a real solution $W = W' \in R^{n\times n}$, $Q \in R^{n\times s}$ for a sufficiently small $\varepsilon > 0$ if and only if for all real ω:

$$2He[(-A^{-1\prime}C\bar{\alpha})'(j\omega I - A)^{-1} 2B] > 0 \tag{22}$$

(positive definite). (22) can readily be transformed into the frequency condition

$$He \, \frac{1}{j\omega} \, \bar{\alpha}[G(j\omega) - G(0)] < 0, \quad \forall\, \omega \in R \tag{23}$$

(negative definite), where

$$G(s) \triangleq C'(sI - A)^{-1} B \tag{24}$$

represents the transfer matrix of the controller's linear dynamic component. Defining $w \triangleq \mathrm{col}\,[w_i;\ i = 1, \dots, r]$; $\mu_d(w) \triangleq \mathrm{col}\left[\frac{\partial \mu}{\partial w_i};\ i = 1, \dots, r\right]$;

$$\zeta_d(w) \triangleq \begin{bmatrix} \frac{\partial \zeta_1}{\partial w_1} & \cdots & \frac{\partial \zeta_1}{\partial w_r} \\ \vdots & \ddots & \vdots \\ \frac{\partial \zeta_s}{\partial w_1} & \cdots & \frac{\partial \zeta_s}{\partial w_r} \end{bmatrix} \in R^{s\times r}$$

and $\eta_d(w)$ similarly the control law (13) becomes

$$u = -[\zeta_d{}'(w)\bar{\alpha} f(\sigma) + 2\eta_d{}'(w) WAz + \mu_d(w)]. \tag{25}$$

Special cases occur for the choices

$$\zeta(w) \equiv 0; \quad \eta(w) = H'w, \quad \text{hence} \quad \eta_d(w) = H', \tag{26}$$

$$\eta(w) \equiv 0; \quad \zeta(w) = Z'w, \quad \text{hence} \quad \zeta_d(w) = Z'. \tag{27}$$

For the choice (26) the controller dynamics simplify to

$$\dot{z} = Az - Bf(C'z) + H'w, \tag{28}$$

$$u = -[2HWAz + \mu_d(w)] \tag{29}$$

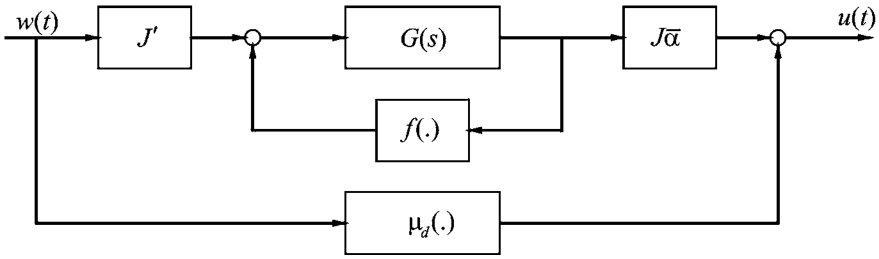

Figure 4.1. Block diagram of the controller (28), (29), (32).

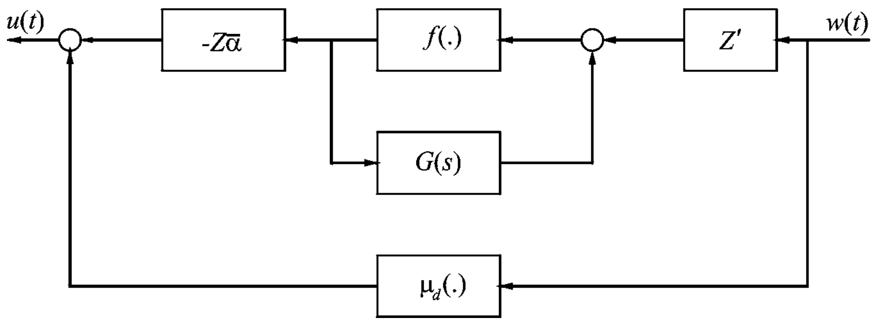

Figure 4.2. Block diagram of the controller (30), (31).

and for the choice (27):

$$\dot{z} = Az - Bf(C'z + Z'w), \tag{30}$$

$$u = -[Z\bar{\alpha}f(C'z + Z'w) + \mu_d(w)]. \tag{31}$$

If we take H of the special form

$$H = JB', \tag{32}$$

then the controller (28), (29) can be represented in the block diagram form of Figure 4.1. The controller (30), (31) has the block diagram representation of Figure 4.2.

5 Discussion

In summary of Section 4 the controller (16), (17), (25) and in particular the controllers of Figures 4.1 and 4.2 ensure closed loop nonoscillatory behaviour provided the EL system (1) is fully damped and the transfer matrix of the controller's linear dynamic component $G(s)$ satisfies condition (23) for some diagonal $\bar{\alpha}$. All other controller components are arbitrary. In the underdamped case nonoscillatory behaviour is still guaranteed if the largest invariant subset $\mathcal{M}$ of the closed loop state space where $\dot{z} \equiv 0$ and $\dot{q}' \frac{\partial \mathcal{F}}{\partial \dot{q}}(\dot{q}) \equiv 0$ consists of the union of the equilibrium points.

Furthermore in (21) we may choose $\varepsilon = 0$, which weakens the negative definiteness condition (23) to negative semidefinite, provided the largest invariant set where $Q'\dot{z} \equiv 0$ and $\dot{q}' \frac{\partial \mathcal{F}}{\partial \dot{q}}(\dot{q}) \equiv 0$ still consists of the union of the equilibrium points.

The condition (23) is relatively mild. In the special case of a single nonlinearity $f(.)$ in the controller structure $(s = 1)$ it simplifies to

$$\frac{\alpha}{\omega} ImG(j\omega) < 0, \quad \forall\, \omega \in R, \tag{33}$$

where $G(s)$ is a scalar transfer function. For example in the second order case $G(s) = 1/(s^2 + as + b)$, (33) is satisfied for $\alpha a > 0$. For $s = 0$, i.e. for controllers without nonlinearities $f(.)$, the conditions (18), (19) simplify to the single condition

$$A'W + WA < 0. \tag{34}$$

It is easy to show that if A has no characteristic values on the imaginary axis there always exists a real symmetric W such that (34) holds.

For nonoscillatory systems the method of the closest unstable equilibrium point is a well known direct method of the Liapunov type for estimating regions of asymptotic stability (RAS) in state space for the system's stable equilibria $\hat{x}_{c,s}$ [5]. The method requires that a global Liapunov function $V_c(x_c) \in C^1$ can be found such that:

1. The associated invariant set $\mathcal{M}$ consists of the union of all equilibrium points.
2. $V_c(x_c)$ possesses an absolute minimum $V_{c,min}$ on the stability boundary of $\hat{x}_{c,s}$. The existence of the minimum is ensured if all solutions of the system remain bounded for $t \geq 0$. Hence the conditions for applicability of the method are exactly those which have been imposed on the control loop in the sections above.

In design problems, once nonoscillatory behaviour has been established the controller's structure must be further specified to implement the control objectives w.r.t. the location of the closed loop equilibria in state space, the linearized system dynamics around the stable equilibria and their RAS. In the next sections we consider the application of EL controllers to an EL system and we present a design example.

6 EL Controllers

In the literature it has been proposed to control EL systems by means of controllers that itself possess an EL structure [6]. Consider an EL controller of the form

$$D_0\ddot{p} + C_0\dot{p} + K_0 p + C_1 f(C_1' p) + \nu(w) = 0, \tag{35}$$

where D_0, K_0 and C_0 are symmetric and positive definite. (35) can be written in the state representation (16), (17) with

$$z \triangleq \begin{bmatrix} p \\ \dot{p} \end{bmatrix}; \quad A \triangleq \begin{bmatrix} 0 & I \\ -D_0^{-1}K_0 & -D_0^{-1}C_0 \end{bmatrix}; \quad B \triangleq \begin{bmatrix} 0 \\ D_0^{-1}C_1 \end{bmatrix};$$

$$C \triangleq \begin{bmatrix} C_1 \\ 0 \end{bmatrix}; \quad \eta(w) \triangleq \begin{bmatrix} 0 \\ -D_0^{-1}\nu(w) \end{bmatrix}; \quad \zeta(w) = 0.$$

Straightforward calculations reveal that $G(s) = G'(s) = {C_1}'[D_0 s^2 + C_0 s + K_0]^{-1} C_1$ such that, observing that $G(0)$ is symmetric and assuming C_1 has full rank s, (23) holds with $\bar{\alpha} = I$ if and only if

$$\frac{1}{2j\omega}\left[(K_0 - D_0\omega^2 + C_0 j\omega)^{-1} - (K_0 - D_0\omega^2 - C_0 j\omega)^{-1}\right]$$
$$= -\{(K_0 - D_0\omega^2)C_0^{-1}(K_0 - D_0\omega^2) + C_0\omega^2\}^{-1} < 0$$

which is true for all $\omega \in R$. (18)−(20) where for simplicity we take $\varepsilon = 0$ yields

$$P = \frac{1}{2}\begin{bmatrix} K_0 & \\ & D_0 \end{bmatrix}; \quad Q = \begin{bmatrix} C_0^{\frac{1}{2}} \\ 0 \end{bmatrix}; \quad p(w) = \begin{bmatrix} \nu(w) \\ 0 \end{bmatrix}$$

while

$$\left(\frac{\partial V}{\partial z}\right)' \dot{z} = -\dot{p}' C_0 \dot{p}. \tag{36}$$

The control law (25) becomes

$$u = -[\nu_d(w)p + \mu_d(w)]. \tag{37}$$

Substituting (36) in the left hand side of (14) shows that (37) renders the closed loop nonoscillatory assuming the controlled EL system (1) satisfies the damping conditions discussed in Section 5.

7 Example

Figure 7.1 displays a simple conceptive example of a one-degree-of-freedom system in its set point equilibrium position $y = 0$. Rescaling time as $\tau = \omega_0 t$; $\omega_0 \triangleq \sqrt{\frac{k}{m}}$ and defining $\zeta \triangleq \frac{c}{2\sqrt{km}}$, $\rho \triangleq \frac{\sqrt{l^2+d^2}}{d}$, $q \triangleq \frac{y}{d}$ and $u \triangleq \frac{f_0}{kd}$ the equation of motion can be written in dimensionless form as:

$$\ddot{q} + 2\zeta\dot{q} + g(q) = u \tag{38}$$

with

$$g(q) \triangleq \left[1 - \frac{\rho}{\sqrt{\rho^2 + q^2 + 2q}}\right](1 + q).$$

There are three open loop equilibria resp. at $q = 0$, $q = -1$ and $q = -2$. A first order controller state equation of the form (16), (17) with $w = q$ reads

$$\dot{z} = -az - f(z) + \eta(q), \quad z \in R, \quad a \neq 0. \tag{39}$$

The frequency condition (33) where $G(s) = \frac{1}{s+a}$ holds for $\alpha > 0$. Now taking $\varepsilon = 0$ some computations yield

$$V_c(x_c) = \frac{1}{2}\dot{q}^2 + \int_0^q g(\theta)\,d\theta + \frac{\alpha a}{2} z^2 + \alpha \int_0^z f(\theta)\,d\theta - \alpha z\eta(q) + \mu(q), \tag{40}$$

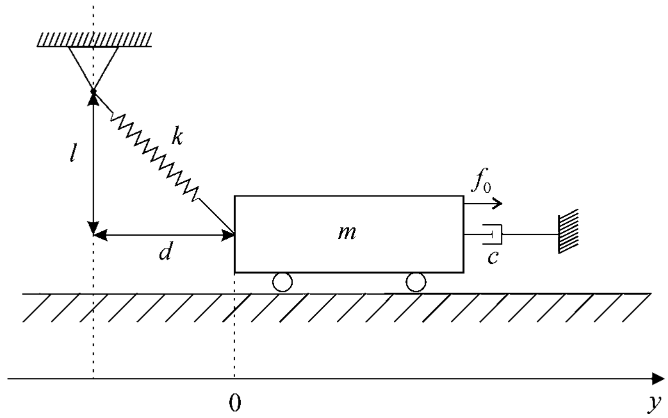

Figure 7.1. A one-degree-of-freedom nonlinear EL system.

where $x_c = [q, \dot{q}, z]'$. Differentation along the solutions of (38), (39) produces

$$\dot{V}_c(x_c) = -2\zeta\dot{q}^2 - \alpha\dot{z}^2 \le 0; \quad \forall\, x_c \in R^3, \tag{41}$$

if according to (13)

$$u = -\frac{\partial V}{\partial q}\,(z, q) = \alpha z \eta_d(q) - \mu_d(q), \tag{42}$$

where the subscript d denotes differentiation w.r.t. q. (41) ensures that in closed loop the state space's largest invariant subset where $\dot{V}_c(x_c) \equiv 0$ consists of the union of the equilibrium points. Next assume that $|\eta(q)|$, $|\eta_d(q)|$ and $|\mu_d(q)|$ are bounded for all $q \in R$ and that

$$\frac{az + f(z)}{z} \ge k_1 > 0; \quad \forall\, z \in R, \quad z \ne 0 \tag{43}$$

with $f(0) = 0$. Then it is an easy exercise to show that:

1. $V_c(x_c)$ is radially unbounded such that the set of the closed loop equilibria is globally convergent.
2. $$\frac{d}{dt}\,z^2 \le 0 \quad \text{for} \quad |z| \ge \frac{|\eta(q)|_{\max}}{k_1} \triangleq n_0 \tag{44}$$

   such that $|z(0)| \le n_0$ implies $|z(t)| \le n_0$ for all $t \ge 0$, hence the control force remains bounded:

   $$|u(t)| \le \alpha n_0 |\eta_d(q)|_{\max} + |\mu_d(q)|_{\max}; \quad \forall\, t \ge 0. \tag{45}$$

Let the desired closed loop equilibria be $x_{c0} = [0, 0, 0]'$ (set point); $x_{c1} = [q_1, 0, 0]'$; $x_{c2} = [q_2, 0, 0]'$, where $q_2 < q_1 < -1$ and let $\Lambda = \{\lambda_i,\ i = 1, \dots, 3\}$ be a selected eigenvalue spectrum in $\{Re\, s < 0\}$ for the linearized closed loop dynamics at x_{c0}. As an example choose

$$\mu_d(q) = \frac{m_1 q + m_2 q^2}{1 + m_3 q^2}; \quad m_3 > 0,$$

$$\eta(q) = \frac{m_4 q (1 - \frac{q}{q_1})(1 - \frac{q}{q_2})}{\sqrt{1 + m_5 q^6}}; \quad m_5 > 0,$$

$$f(z) = \gamma z^3; \quad \gamma > 0$$

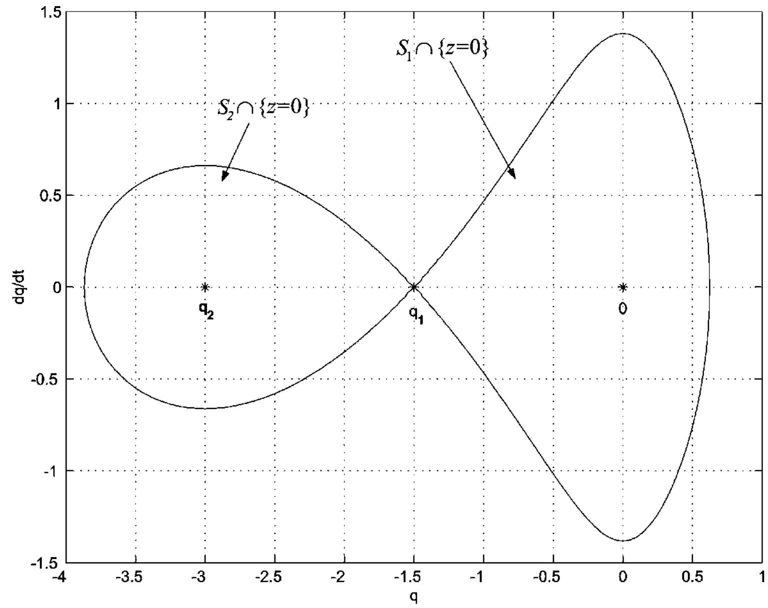

Figure 7.2. Intersections of the estimated stability regions S_1 and S_2 with the plane $\{z=0\}$. Parameter values: $\rho = 2,\ q_1 = -1.5,\ q_2 = -3,\ \Lambda = \{-1, -1 + j\sqrt{3}, -1 - j\sqrt{3}\},\ a = 3,\ m_1 = 5.75,\ m_2 = 3.6238,\ m_3 = 3.3861$.

which implies that $k_1 = a > 0$. Now a straightforward analysis reveals that:

1. For $\alpha > 0$ and sufficiently small there are no other closed loop equilibria besides x_{c0} (of index 0), x_{c1} (of index 1) and x_{c2} (of index 0).
2. Λ, q_1 and q_2 can be arbitrarily assigned by suitabl tuning the parameters a and $m_1 \to m_4$.

In addition to α the remaining free control parameters are m_5 and γ. Their choice influences the upper bound of the control force $|u(t)|_{\max}$, the extent of the set point's region of attraction in state space and the linearized dynamics at x_{c1} and x_{c2}. The method of the closest unstable equilibrium point produces the set

$$S \triangleq \{x_c \in R^3;\ V_c(x_c) < V_c(x_{c1})\}$$

which consists of two disjoint subsets $S_1 \ni x_{c0}$ and $S_2 \ni x_{c2}$. These constitute estimated regions of attraction for x_{c0} and x_{c2}. The control parameters α, m_5 and γ do not affect the intersections of S_1 and S_2 with the plane $\{z = 0\}$ (Figure 7.2), but they bear an influence on the extent of the stability regions in the z-direction (Figure 7.3).

8 Conclusion: Extensions and Further Work

We have derived sufficient conditions for a class of nonlinear feedback controllers for EL systems to render the closed loop nonoscillatory. The obtained results can be extended in several ways. As to the controlled system, other classes of dissipative processes can be considered possessing various types of nonlinear components. Noncollocal control of EL

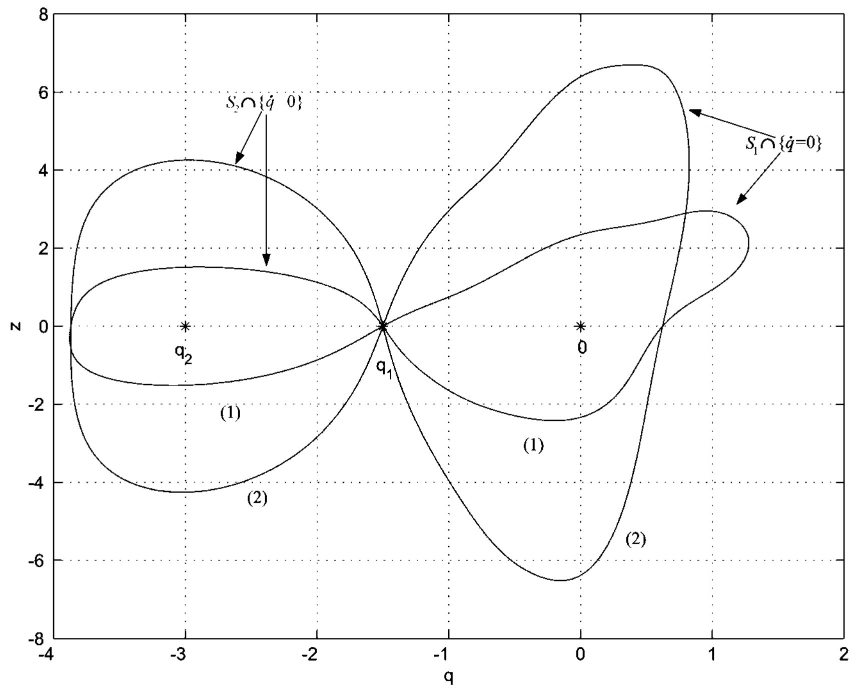

Figure 7.3. Intersections of the estimated stability regions S_1 and S_2 with the plane $\{\dot{q} = 0\}$ for the parameter values of Figure7.2 and for: (1) $\alpha = 0.025$, $m_4 = 23.6643$, $\gamma = 4$, $m_5 = 0.5$; (2) $\alpha = 0.002$, $m_4 = 83.6660$, $\gamma = 1$, $m_5 = 5$.

systems may be studied. As to the structure of the controller the frequency condition on its dynamic component may be further weakened at the expense of imposing some restrictions on the nonlinear amplifier characteristics. For example we may consider monotonous or slope restricted nonlinearities. In view of the fact that wide classes of neural networks have a state description of the form (16), (17), possible applications include neural control of nonlinear systems. Research will be conducted to incorporate the proposed theory in practical controller design procedures and to analyse specific applications.

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