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(G'/G)-Expansion Method and Weierstrass Elliptic Function Method Applied to Coupled Wave Equation

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Abstract: This paper deals with the exact solutions of a nonlinear coupled wave equation. The (G'/G)-expansion method has been applied to derive kink solutions and singular wave solutions. The restrictions on the coefficients of the governing equations have also been investigated. Solitary wave solutions have also been derived for this system of equations using the Weierstrass elliptic function method.

Keywords: *(G'/G)-expansion method; coupled wave equation; kink wave solutions; singular wave solutions; solitary wave solutions; Jacobi and Weierstrass elliptic functions.*

Mathematics Subject Classification (2010): 74J35, 34G20, 93C10.

1 Introduction

Nonlinear evolution equations (NLEEs) govern several physical phenomena which appear in various branches of science and engineering [1–5]. Exact solutions of NLEEs shed more light on the various aspects of the problem, which, in turn, leads to the applications. Several methods such as the tanh method [6–11], exponential function method [12], Jacobi elliptic function (JEF) method [13–16], mapping methods [17–22], Hirota bilinear method [23, 24] and trigonometric-hyperbolic function methods [25–27] have been applied in the last few decades and the results have been reported. Also, many physical phenomena have been governed by systems of partial differential equations (PDEs) and there have been significant contributions in this area [28, 29].

In this paper, we use the (G'/G)-expansion method [30–34] to find some exact solutions for a nonlinear coupled wave equation [35]. The paper is organized as follows. In Section 2, we give a mathematical analysis of the (G'/G)-expansion method, in Section 3, we find kink solutions and singular wave solutions of the nonlinear coupled wave equation, in Section 4, we use the Weierstrass elliptic function (WEF) method [36] to derive SWSs of the system of equations, in Section 5 we write down the conclusion.

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2 The (G'/G)-Expansion Method

Consider the nonlinear partial differential equation (PDE)

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, ...) = 0, \tag{1}$$

where $u(x,t)$ is an unknown function, P is a polynomial in $u = u(x,t)$ and its various partial derivatives. The traveling wave variable $\xi = x - ct$ reduces the PDE (1) to the ordinary differential equation (ODE)

$$P(u, -cu', u', -c^2u'', -cu'', u'', ...) = o, \tag{2}$$

where $u = u(\xi)$ and $'$ denotes differentiation with respect to ξ.

We suppose that the solution of equation (2) can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i \left(\frac{G'}{G}\right)^i, \;\; a_m \neq 0, \tag{3}$$

where $a_i (i = 0, 1, 2, ..)$ are constants. Here, G satisfies the second order linear ODE

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \tag{4}$$

with λ and μ being constants. The positive integer m can be determined by a balance between the highest order derivative term and the nonlinear term appearing in equation (2). By substituting equation (3) into equation (2) and using equation (4), we get a polynomial in G'/G. The coefficients of various powers of G'/G give rise to a set of algebraic equations for a_i $(i = 0, 1, 2, ..., m)$, λ and μ.

The general solution of equation (4) is a linear combination of sinh and cosh or of sine and cosine functions if $\Delta = \lambda^2 - 4\mu > 0$ or $\Delta = \lambda^2 - 4\mu < 0$, respectively. In this paper we consider only the first case and so,

$$G(\xi) = e^{-\lambda\xi/2} \left(C_1 \sinh(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi) + C_2 \cosh(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi) \right), \tag{5}$$

where C_1 and C_2 are arbitrary constants.

3 A Coupled Wave Equation

Consider the system of PDEs

$$u_t + \alpha v^2 v_x + \beta u^2 u_x + \eta u u_x + \gamma u_{xxx} = 0, \tag{6}$$

$$v_t + \sigma (uv)_x + \epsilon v v_x = 0, \tag{7}$$

where, $\alpha, \beta, \eta, \gamma, \sigma$ and ϵ are constants.

We seek TWSs of equations (6) and (7) in the form $u = u(\xi)$, $\quad v = v(\xi)$, $\quad \xi = x - ct$. Then equations (6) and (7) give

$$-cu' + \alpha v^2 v' + \beta u^2 u' + \eta u u' + \gamma u''' = 0, \tag{8}$$

$$-cv' + \sigma(uv)' + \epsilon vv' = 0. \tag{9}$$

Integrate equation (9) with respect to ξ

$$-cv + \sigma(uv) + \frac{\epsilon}{2}v^2 = k, \tag{10}$$

where k is the integration constant. Dividing equation (10) by v, we obtain

$$-c + \sigma u + \frac{\epsilon}{2}v = \frac{k}{v}. \tag{11}$$

So, for the solutions to be uniformly valid, the integration constant k should be set equal to 0. Therefore, equation (11) can be written as

$$v = \frac{2(c - \sigma u)}{\epsilon}. \tag{12}$$

Substituting equation (12) into equation (8), we obtain

$$-cu' - \frac{8\alpha\sigma}{\epsilon^3}(c^2 - 2c\sigma u + \sigma^2u^2)u' + \beta u^2u' + \eta uu' + \gamma u''' = 0. \tag{13}$$

Integrating equation (13) with respect to ξ and assuming the boundary conditions $u, u', u'' \longrightarrow 0$ as $|\xi| \longrightarrow \infty$, we have

$$\gamma u'' - \left\{c + \frac{8\alpha\sigma c^2}{\epsilon^3}\right\}u + \left\{\frac{\eta}{2} + \frac{8\alpha\sigma^2 c}{\epsilon^3}\right\}u^2 + \left\{\frac{\beta}{3} - \frac{8\alpha\sigma^3}{3\epsilon^3}\right\}u^3 = 0. \tag{14}$$

We rewrite equation (14) as

$$\gamma u'' + Au + Bu^2 + Cu^3 = 0, \tag{15}$$

where

$$A = -\left\{c + \frac{8\alpha\sigma c^2}{\epsilon^3}\right\}, \quad B = \left\{\frac{\eta}{2} + \frac{8\alpha\sigma^2 c}{\epsilon^3}\right\}, \quad C = \left\{\frac{\beta}{3} - \frac{8\alpha\sigma^3}{3\epsilon^3}\right\}. \tag{16}$$

With the change of variable $w = u + \delta$, equation (15) can be reduced to

$$\gamma w'' + c_1 w + c_2 w^3 + c_3 = 0, \tag{17}$$

where

$$\delta = -\frac{B}{3C}, \quad c_1 = \frac{3AC - B^2}{3C}, \quad c_2 = C, \quad c_3 = \frac{2B^3 - 9ABC}{27C^2}. \tag{18}$$

Assuming the expansion $w(\xi) = \sum_{i=0}^{m} a_i \left(\frac{G'}{G}\right)^i$, $a_m \neq 0$ in equation (17) and balancing the nonlinear term and the derivative term, we get $m + 2 = 3m$ so that $m = 1$.

So, we assume the solution of equation (17) in the form

$$w(\xi) = a_0 + a_1\left(\frac{G'}{G}\right), \ a_1 \neq 0. \tag{19}$$

So, we can obtain

$$w'(\xi) = -a_1 \left(\frac{G'}{G}\right)^2 - \lambda a_1 \left(\frac{G'}{G}\right) - \mu a_1, \tag{20}$$

$$w''(\xi) = 2a_1 \left(\frac{G'}{G}\right)^3 + 3a_1\lambda \left(\frac{G'}{G}\right)^2 + (a_1\lambda^2 + 2a_1\mu) \left(\frac{G'}{G}\right) + a_1\lambda\mu, \tag{21}$$

$$w^3(\xi) = a_1^3 \left(\frac{G'}{G}\right)^3 + 3a_0a_1^2 \left(\frac{G'}{G}\right)^2 + 3a_0^2a_1 \left(\frac{G'}{G}\right) + a_0^3. \tag{22}$$

Now, substituting equations (19), (21) and (22) into equation (17) and collecting the coefficients of $\left(\frac{G'}{G}\right)^i$, $i = 0, 1, 2, 3$, we get

$$\gamma a_1\lambda\mu + c_1a_0 + c_2a_0^3 + c_3 = 0, \tag{23}$$

$$\gamma a_1\lambda^2 + 2\gamma a_1\mu + c_1a_1 + 3c_2a_0^2a_1 = 0, \tag{24}$$

$$3a_1\lambda\gamma + 3a_0a_1^2c_2 = 0, \tag{25}$$

$$2\gamma a_1 + c_2a_1^3 = 0. \tag{26}$$

From equation (26), we get

$$a_1 = \pm\sqrt{-\frac{2\gamma}{c_2}}. \tag{27}$$

Equation (25) leads to

$$a_0 = \pm\frac{1}{2}\lambda\sqrt{-\frac{2\gamma}{c_2}}. \tag{28}$$

When $\mu = 0$ in equation (24), we get $\lambda = \pm\sqrt{\frac{2c_1}{\gamma}}$, and when $\lambda = 0$, we get $\mu = -\frac{c_1}{2\gamma}$. In both cases, $\Delta = \lambda^2 - 4\mu = \frac{2c_1}{\gamma}$. Equation (23) gives a constraint condition on the coefficients in the governing equation.

Case 1: $\mu = 0$, $\lambda = \sqrt{\frac{2c_1}{\gamma}}$.

$$u_1(x,t) = \pm\sqrt{-\frac{c_1}{c_2}} \left[1 + \frac{(C_1 - C_2)\left(1 - \tanh\frac{1}{2}\sqrt{\frac{2c_1}{\gamma}}(x-ct)\right)}{C_1\tanh\frac{1}{2}\sqrt{\frac{2c_1}{\gamma}}(x-ct) + C_2}\right] + \frac{B}{3C}. \tag{29}$$

Case 2: $\mu = 0,\ \lambda = -\sqrt{\dfrac{2c_1}{\gamma}}$.

$$u_2(x,t) = \pm\sqrt{-\frac{c_1}{c_2}}\left[1 + \frac{(C_1+C_2)\left(1-\tanh\frac{1}{2}\sqrt{\frac{2c_1}{\gamma}}(x-ct)\right)}{C_2 - C_1\tanh\frac{1}{2}\sqrt{\frac{2c_1}{\gamma}}(x-ct)}\right] + \frac{B}{3C}. \tag{30}$$

Case 3: $\lambda = 0,\ \mu = -\dfrac{c_1}{2\gamma}$.

$$u_3(x,t) = \pm\sqrt{-\frac{c_1}{c_2}}\left[\frac{C_1 + C_2\tanh\frac{1}{2}\sqrt{\frac{2c_1}{\gamma}}(x-ct)}{C_1\tanh\frac{1}{2}\sqrt{\frac{2c_1}{\gamma}}(x-ct) + C_2}\right] + \frac{B}{3C}. \tag{31}$$

In all three cases, γ and c_1 should have the same signs and c_2 should be of the opposite sign and $C_1 \neq \pm C_2$.

Figure 1 and Figure 2 represent the solutions given by equation (29).

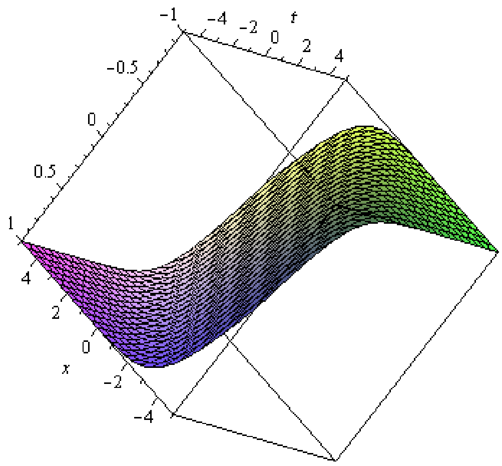

Figure 1: The solution for $u(x,t)$, $C_1 = 0, C_2 = 1$.

Using equation (12), we can write down the corresponding solutions $v_1(x,t)$, $v_2(x,t)$ and $v_3(x,t)$.

4 Weierstrass Elliptic Function Solutions of the Coupled Wave Equation

The Weierstrass elliptic function (WEF) $\wp(\xi; g_2, g_3)$ with invariants g_2 and g_3 satisfy

$$\wp'^2 = 4\wp^3 - g_2\wp - g_3, \tag{32}$$

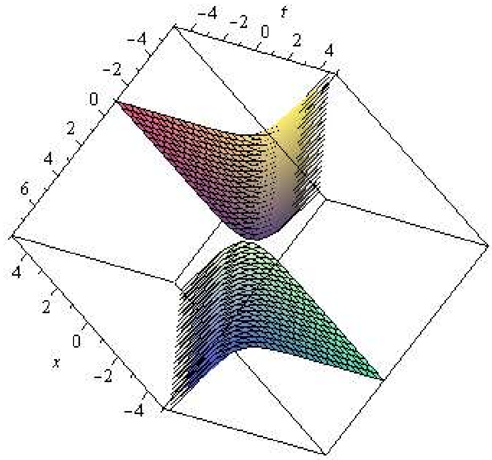

Figure 2: The solution for $u(x,t)$, $C_1 = 1, C_2 = 0$.

where g_2 and g_3 are related by the inequality

$$g_2^3 - 27g_3^2 > 0. \tag{33}$$

The WEF $\wp(\xi)$ is related to the JEFs by the following relations:

$$\mathrm{sn}(\xi) = [\wp(\xi) - e_3]^{-1/2}, \tag{34}$$

$$\mathrm{cn}(\xi) = \left[\frac{\wp(\xi) - e_1}{\wp(\xi) - e_3}\right]^{1/2}, \tag{35}$$

$$\mathrm{dn}(\xi) = \left[\frac{\wp(\xi) - e_2}{\wp(\xi) - e_3}\right]^{1/2}, \tag{36}$$

where e_1, e_2, e_3 satisfy

$$4z^3 - g_2 z - g_3 = 0 \tag{37}$$

with

$$e_1 = \frac{1}{3}(2 - m^2),\ \ e_2 = \frac{1}{3}(2m^2 - 1),\ \ e_3 = -\frac{1}{3}(1 + m^2). \tag{38}$$

From equation (38), one can see that the modulus m of the JEF and the $e's$ of the WEF are related by

$$m^2 = \frac{e_2 - e_3}{e_1 - e_3}. \tag{39}$$

We consider the ODE of order $2k$ given by

$$\frac{d^{2k}\phi}{d\xi^{2k}} = f(\phi; r+1), \tag{40}$$

where $f(\phi; r+1)$ is an $(r+1)$ degree polynomial in ϕ. We assume that

$$\phi = \gamma Q^{2s}(\xi) + \mu \tag{41}$$

is a solution of equation (40), where γ and μ are arbitrary constants and $Q^{(2s)}(\xi)$ is the $(2s)^{\text{th}}$ derivative of the reciprocal Weierstrass elliptic function (RWEF) $Q(\xi) = \dfrac{1}{\wp(\xi)}$, $\wp(\xi)$ being the WEF.

It can be shown that the $(2s)^{\text{th}}$ derivative of the RWEF $Q(\xi)$ is a $(2s+1)$ degree polynomial in $Q(\xi)$ itself. Therefore, for ϕ to be a solution of equation (40), we should have the relation

$$2k - r = 2rs. \tag{42}$$

So, it is necessary that $2k \geq r$ for us to assume a solution in the form of equation (41). But this is in no way a sufficient condition for the existence of the PWS in the form of equation (41).

Now, we shall search for the WEF solutions of equation (17). We introduce a restriction on the coefficients in the form $2B^2 = 9AC$, so that equation (17) reduces to

$$\gamma w'' + c_1 w + c_2 w^3 = 0, \tag{43}$$

where $c_1 = -\dfrac{1}{2}A,\ c_2 = C$.

For a solution of equation (43) in the form of equation (41), we should have $r = 2$ and $k = 1$ so that $s = 0$. So, our solution will be

$$u(\xi) = \frac{\tau}{\wp(\xi)} + \zeta. \tag{44}$$

Substituting equation (44) into equation (43) and equating the coefficients of like powers of $\wp(\xi)$ to zero, we obtain

$$\wp^3(\xi): \quad 2\gamma\tau - \frac{1}{2}A\zeta + C\zeta^3 = 0, \tag{45}$$

$$\wp^2(\xi): \quad -\frac{1}{2}A\tau + 3C\tau\zeta^2 = 0, \tag{46}$$

$$\wp(\xi): \quad -\frac{3}{2}\gamma\tau g_2 + 3C\tau^2\zeta = 0, \tag{47}$$

$$\wp^0(\xi): \quad -2\gamma\tau g_3 + C\tau^3 = 0. \tag{48}$$

From equations (46) through (48), it can be found that

$$\tau = \pm\sqrt{\frac{2\gamma g_3}{C}}, \tag{49}$$

$$\zeta = \pm\sqrt{\frac{A}{6C}}, \tag{50}$$

$$g_2 = 2\sqrt{\frac{Ag_3}{3\gamma}}. \tag{51}$$

From equations (49) through (51), one can conclude that if $g_3 > 0$, A, C and γ should all be of the same signs, whereas for $g_3 < 0$, A and C should be of the same signs and γ should be of the opposite sign.

Equation (45) leads us to the value of g_3 given by

$$g_3 = \frac{A^3}{432\gamma^3}. \tag{52}$$

The condition $g_2^3 - 27g_3^2 > 0$ gives the relation

$$\frac{8}{9} > \frac{3}{4}, \tag{53}$$

which is remarkably true for any value of the coefficients in the governing equation.

The equations (34) through (36) will give rise to the same PWS of equation (43) which can be obtained using equation (44) with the help of equation (38). Thus the PWS of equation (43) in terms of JEFs can be written as

$$u(\xi) = \frac{\tau\,\mathrm{sn}^2(\xi)}{1 - \frac{1}{3}(1+m^2)\mathrm{sn}^2(\xi)} + \zeta. \tag{54}$$

As $m \to 1$, the SWSs of the coupled wave equation given by equations (6) and (7) with the restriction $2B^2 = 9AC$ are

$$u(x,t) = \frac{B}{3C} + \frac{\tau\tanh^2(x-ct)}{1 - \frac{2}{3}\tanh^2(x-ct)} + \zeta, \tag{55}$$

$$v(x,t) = \frac{2c}{\epsilon} - \frac{2\sigma}{\epsilon}\left[\frac{B}{3C} + \frac{\tau\tanh^2(x-ct)}{1 - \frac{2}{3}\tanh^2(x-ct)} + \zeta\right], \tag{56}$$

where τ and ζ are given by equations (49) and (50).

5 Conclusions

The (G'/G)-expansion method has been applied to a nonlinear coupled wave equation. The kink wave solution and the singular wave solution have been graphically illustrated. It was found that there are some restrictions on the coefficients in the governing equation for the solutions in terms of hyperbolic functions to exist. The WEF method has also been applied to the system of equations to derive SWSs. The condition $g_2^3 - 27g_3^2 > 0$ was found to be identically satisfied, which is a remarkable result and has never been reported in the literature. We intend to apply these methods for higher order and higher dimensional PDEs of physical interest.

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