

## Oscillation and Nonoscillation for Caputo–Hadamard Impulsive Fractional Differential Equations

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**Abstract:** In this paper, the concept of the upper and lower solutions method combined with the fixed point theorem is used to investigate the existence of oscillatory and nonoscillatory solutions for a class of initial value problems for Caputo–Hadamard impulsive fractional differential equations.

**Keywords:** impulsive fractional differential equations; Caputo–Hadamard fractional derivative; fixed point; upper solution; lower solution, oscillation, nonoscillation.

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## 1 Introduction

Fractional differential equations and integrals are valuable tools in the modeling of many phenomena in various fields of science and engineering. Indeed, there are numerous applications in viscoelasticity, electrochemistry, control, porous media, electromagnetism, etc. In the monographs [1,3,4,11,12,15], we can find the mathematical background and various applications of fractional calculus. Recently, many researchers studied different fractional problems involving the Riemaan-Liouville, Caputo and Hadamard derivatives; see, for example, the papers [2,17]. Sufficient conditions for the oscillation of solutions of differential equations are given in [9,14,16].

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