



Extensions of Schauder's and Darbo's Fixed Point Theorems

Zhaocai Hao^{1,2}, Martin Bohner^{1*} and Junjun Wang²

¹ *Missouri S&T, Rolla, MO 65409, USA*

² *Qufu Normal University, Qufu 273165, Shandong, China*

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Abstract: In this paper, some new extensions of Schauder's and Darbo's fixed point theorems are given. As applications of the main results, the existence of global solutions for first-order nonlinear integro-differential equations of mixed type in a real Banach space is investigated.

Keywords: *nonlinear integro-differential equation; Darbo fixed point theorem; Schauder fixed point theorem; Kuratowski measure of noncompactness.*

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1 Introduction

It is well known that the following two fixed points are very important.

Theorem 1.1 (Schauder's fixed point theorem) *Let Ω be a nonempty, bounded, closed, and convex subset of a Banach space E . Then each continuous and compact map $T : \Omega \rightarrow \Omega$ has at least one fixed point in Ω .*

The Schauder fixed point theorem plays an important role in nonlinear analysis. In 1955, Darbo [9] proved a fixed point property for set-contraction on a closed, bounded and convex subset of Banach spaces in terms of the measure of noncompactness, which was first defined by Kuratowski [17].

* Corresponding author: <mailto:bohner@mst.edu>