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Particle Distributions in Nucleation Lattice Models: A Matrix Approach

O. Gutiérrez *

Universidad Autnoma de Barcelona – Campus de Bellaterra – 08193 Spain

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Abstract: In this paper we use a matrix approach to investigate the distribution of particles in nucleation coalescence models with discrete lattices, both in the irreversible coagulation case and in the reversible one. In the irreversible case ($A + A \to A$), the evolution of the particle distribution is described by means of a simple recursive procedure. In two particular cases the model is analytically solvable: with high density and particles that always fuse into one, and in the case of constant density. In the reversible case ($A + A \rightleftharpoons A$) offspring production is allowed, and the system can reach a stationary distribution, which is jointly calculated with the equilibrium density. The particular case, in which meeting particles react with probability one, admits an exact solution.

Keywords: *coalescence models; Markov chain; exponential matrix; Poisson distribution; phase transition.*

Mathematics Subject Classification (2010): 82Cxx, 15A16, 15A18, 60J10.

1 Introduction

In the last decades, diffusion-controlled coalescence processes have attracted much research interest [1], [2], [3], [4], [5], [6] (see [7] and [8] for literature reviews). The models of these processes are applied to the analysis of phenomena involving particles in a solid, chemical species which randomly hop and react with adjacent ones, or non-equilibrium processes ranging from fluorescence to explosions. This kind of models is increasingly being used in biology, chemistry, genetics, sociology or finance, see [9], [10], [11] and [12], in which variations of the Ising model are used. We apply a simple matrix approach to the analysis of one-dimension coalescence models that usually require sophisticated mathematical tools (or Monte-Carlo simulations) to be solved.

* Corresponding author: mailto:oscar.gutierrez@uab.es