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Optimal Voltage Controller using T-S Fuzzy Model for Multimachine Power Systems

A. Abbadi, F. Hamidia, A. Morsli and A. Tlemcani *

Electrical Engineering and Automatic Research Laboratory LREA, Electrical Engineering Department, University of Medea, Algeria

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Abstract: This paper presents an LMI approach to optimal fuzzy control based on the quadratic performance function to enhance the transient stability and achieve voltage regulation for multimachine power systems. First, the dynamic model of the power system has been modeled by Takagi-Sugeno fuzzy systems using the method of sum of products of linearly independent functions. The optimal fuzzy controller proposed is designed by solving the minimization problem that minimizes the upper bound of a given quadratic performance function. The stability conditions are represented in terms of LMIs. The proposed controller is applied to a two-machine three-bus power system. Simulation results illustrate the performance of the developed approach regardless of the system operating conditions.

Keywords: *multimachine power system; T-S fuzzy model; optimal fuzzy control; Lyapunov stability; linear matrix inequalities (LMI).*

Mathematics Subject Classification (2010): 03B52, 93C42, 94D05.

1 Introduction

System stability is the most important issue for power systems; traditionally, transient and voltage instability have been the most widespread stability problems. They concern the maintenance of the synchronism between generators as well as a steady acceptable voltage under normal operating and disturbed conditions.

Modern power systems are highly complex and nonlinear, and their operating conditions can vary over a wide range, therefore, the nonlinear characteristics of the power system and, hence, the nonlinear dynamic model of the system should be used in the analysis of transient stability and voltage regulation.

* Corresponding author: mailto:h_tlemcani@yahoo.fr

One of many design techniques developed for modeling and control of nonlinear systems is the Takagi-Sugeno (T-S) one [1–7]. The approach mainly consists of three stages. The first stage is the fuzzy modeling for nonlinear controlled objects. There are two major ways in fuzzy modeling. One is the fuzzy model identification [2,3] using input-output data. The other is the fuzzy model construction (fuzzy IF-THEN rules) based on the idea of sector nonlinearity. The second stage is the fuzzy control rule derivation that mirrors the rule structure of a fuzzy model. It is realized by the so-called parallel distributed compensation (PDC) [4–6]. The third stage is the fuzzy controller design, i.e., the determination of feedback gains stated in terms of linear matrix inequalities (LMI) [5]; the stability is investigated using the quadratic Lyapunov function. Generally, such a design focuses on the stability issue only and does not satisfy certain performance criteria and constraints in an optimal fashion.

In the control design, it is often of interest to synthesize a controller to satisfy, in an optimal fashion, certain performance criteria and constraints in addition to stability [5].

In the linear case, the optimization problem is resolved by determining an optimal feedback of a Ricatti equation [8–10] . This type of controller is known under the name of a linear quadratic regulator problem (LQR). For the nonlinear systems, the problem requires the resolution of the Hamilton-Jacobi-Bellman (HJB) equation which represents a partial derivative equation [11, 12].

In the field of the power system stability, Kharaajoo in [13] has used an aproximative solution of the HJB equation to enhance the transient stability and achieve voltage regulation of a single-machine infinite-bus power system. The global control law is represented by the average of two control laws weighted by a sensitivity indicator such that the closed-loop power system is transiently stable when subjected to a fault, and restores the steady pre-fault voltage value after the disturbance. The analytical solution of this HJB equation was very difficult to be found, so an approximate method using the Taylor series expansion is used.

As the Takagi-Sugeno (T-S) fuzzy system is an efficient approach to model the nonlinear systems, Tanaka [5] proposed an alternative approach to nonlinear optimal control based on fuzzy logic. The optimal fuzzy control methodology presented is designed by solving a minimization problem that minimizes the upper bound of a given quadratic performance function. In strict sense, this approach is a suboptimal design. One of the advantages of this methodology is that the design conditions are represented in terms of LMIs.

This paper presents an optimal fuzzy controller design via convex optimization techniques based on LMIs to enhance the transient stability and achieve voltage regulation of multimachine power systems. The DFL technique has been used to linearize and decouple a nonlinear n-machine power system to n independent DFL compensated models. Then these compensated models are described by continuous-time T-S models. The fuzzy system is stabilized by the PDC fuzzy controller based on the minimization of the upper bound of a quadratic performance function.

To begin with, in Section 2, the background materials concerning the T-S fuzzy model and model-based fuzzy controller are introduced. In Section 3, the optimal fuzzy controller design is presented. The equivalent T-S fuzzy model of the multimachine power system is developed in Section 4. In Section 5, the control scheme proposed is implemented in a two-machine three-bus power system and simulation results are provided to demonstrate the performance of the proposed optimal voltage controller. Finally, conclusions are drawn in Section 6.

2 T-S Fuzzy Model and Control

2.1 T-S fuzzy model

A nonlinear system can be approximated by a T-S fuzzy model. The T-S model consists of a set of IF-THEN rules. Each rule represents the local linear input-output relation of the nonlinear system and has the following form:

Plant Rule i:

$$\begin{cases} \text{If } z_1(t) \text{ is } M_{i,1} \text{.... and } z_p(t) \text{ is } M_{i,p}, \\ \text{then } \dot{x_i} = A_i x(t) + B_i u(t), i = 1, 2, ..., r. \end{cases} \tag{1}$$

Here $z(t) = \{z_1(t), ..., z_p(t)\}$ are known as premise variables, i.e., the nonlinear terms appeared in the system equations. Those premise variables are usually functions of the state variables. Also, $M_{i,j}$ is the fuzzy set, r is the number of model rules, A_i and B_i are the system and input matrices, respectively. It is assumed that (A_i,B_i) is a controllable pair. Also, x(t) is the system state vector, and u(t) is the input vector. The overall system dynamics is then described as

$$\dot{x} = \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(t)). \tag{2}$$

2.2 T-S model-based fuzzy control

The concept of PDC, following the terminology of [5], is utilized to design fuzzy state-feedback controllers on the basis of the T-S fuzzy models (1). In the PDC design, each control rule is designed from the corresponding rule of a T-S fuzzy model. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts. For the fuzzy models (1), we construct the following fuzzy controller via the PDC:

Control rule i:

$$\begin{cases} \text{If } z_1(t) \text{ is } M_{i,1} \text{....... and } z_p(t) \text{ is } M_{i,p}, \\ \text{then } u(t) = -K_i x(t), i = 1, 2, ..., r, \end{cases} \tag{3}$$

where K_i is a linear state feedback gain for the i-th subsystem. The overall fuzzy controller is represented by

$$u(t) = -\sum_{i=1}^{r} K_i x(t), \quad i = 1, 2, ..., r. \tag{4}$$

Substituting equation(4) into equation (2), the fuzzy control system (FCS) can be represented by (closed-loop)

$$u(t) = -\sum_{i=1}^{r} \sum_{i=j}^{r} h_i(z(t)) h_j(z(t))(A_i - B_i K_j) x(t). \tag{5}$$

3 Optimal Fuzzy Controller Design

This section presents an optimal fuzzy controller design which consists in the determination of the control laws that minimize the upper bound of the following quadratic performance function:

$$J = \int_0^\infty (x^T(t)Wx(t) + u^T(t)Ru(t))dt, \tag{6}$$

where W and R are assumed to be a semi-positive definite matrix and a positive definite matrix, respectively [5]. Weighting matrices W and R are important components in the optimizing process of the fuzzy controller since they have great influences on system performance. Sufficient optimality conditions derived by Tanaka [5] for ensuring stability of (5) are given as follows

Theorem 3.1 *[5] The feedback gains to minimize the upper bound of the performance function can be obtained by solving the following LMIs. From the solution of the LMIs, the feedback gains are obtained as*

$$K_i = Y_i Q^{-1}$$

for all i. Then, the performance function satisfies $J < x^T(0)Px(0) < \gamma$,

minimize γ
$Q, Y_1, ...Y_r$ *subject to*

$$Q > 0,$$

$$\begin{pmatrix} 1 & x^T(0) \\ x(0) & Q \end{pmatrix} > 0, \tag{7}$$

$$\begin{pmatrix} QA_i^T + A_iQ - Y_i^T B_i^T - B_iY_i & Q\sqrt{W} & (-Y_i^T)\sqrt{R} \\ \sqrt{W}Q & -I & 0 \\ \sqrt{R}(-Y_i) & 0 & -I \end{pmatrix} < 0, \tag{8}$$

$$\begin{pmatrix} T & Q\sqrt{W} & \frac{\sqrt{2}}{2}(-Y_i^T)\sqrt{R} & \frac{\sqrt{2}}{2}(-Y_j^T)\sqrt{R} \\ \sqrt{W}Q & -I & 0 & 0 \\ \sqrt{R}(-Y_i)\frac{\sqrt{2}}{2} & 0 & -I & 0 \\ \sqrt{R}(-Y_j)\frac{\sqrt{2}}{2} & 0 & 0 & -I \end{pmatrix} < 0, \tag{9}$$

where $T = (\frac{QA_i^T + A_iQ}{2}) + \frac{QA_i^T + A_jQ}{2} - \frac{Y_i^T B_j^T + B_jY_i}{2} - \frac{Y_j^T B_i^T + B_iY_j}{2}$.

4 T-S Fuzzy Model of Power System

4.1 Dynamic model of power system

As the global control objective in this paper is to maintain the transient stability and achieve proper post-fault voltage of the multimachine power system, the dynamic model of the i-th generator adopted is given by the following equations:

$$\begin{cases} \Delta\dot{V}_{t_i}(t) = f_{i_1}(t)\Delta\omega_i(t) - \frac{f_{i_2}(t)}{T'_{d0i}}\Delta P_{ei}(t) + \frac{f_{i_2}(t)}{T_{d0i}}v_{fi}(t), \\ \Delta\dot{\omega}_i(t) = -\frac{D_i}{2H_i}\Delta\omega_i(t) - \frac{\omega_0}{2H_i}\Delta P_{ei}(t), \\ \Delta\dot{P}_{ei}(t) = -\frac{1}{T'_{d0i}}\Delta P_{ei}(t) + \frac{1}{T'_{d0i}}v_{fi}(t). \end{cases} \tag{10}$$

where

$$u_{fi}(t) = \frac{1}{k_{ci}Iqi(t)}(v_{fi}(t) - T'_{d0i}E'_{qi}\dot{I}_{qi}(t) + P_{mi}) + \frac{1}{k_{ci}}((x_{di} - x'_{di})I_{di}(t)), \quad (11)$$

is the direct feedback linearization (DFL)control law and $v_{fi}(t)$ is the feedback control law

$$v_{fi}(t) = -k_{v_i}\Delta V_i(t) - k_{\omega_i}\Delta\omega_i(t) - k_{p_{ei}}\Delta P_{ei}(t) \quad (12)$$

and

$$\begin{cases} \Delta V_{t_i}(t) = V_{ti} - V_{ti0}, \\ \Delta\omega_i(t) = \omega_i - \omega_0, \\ \Delta P_{ei}(t) = P_{ei}(t) - P_{mi}, \end{cases} \quad (13)$$

$$\Delta V_{t_i}(t) = V_{ti} - V_{ti0}, \Delta\omega_i(t) = \omega_i - \omega_0, \Delta P_{ei}(t) = P_{ei}(t) - P_{mi}, \quad (14)$$

$$f_{i_1} = -\frac{(1 + x'_{di}B_{ii})(-E'^2_{qi}(t)B_{ii} - Q_{ei}(t)V_{ti}(t))}{V_{ti}(t)Ia_{qi}(t)} - \frac{x'_{di}(1 + x'_{di}B_{ii})P_{ei}(t)}{V_{ti}}, \quad (15)$$

$$f_{i_2} = -\frac{(1 + x'_{di}B_{ii})V_{ti}(t)}{V_{ti}I_{qi}(t)}, \quad (16)$$

where $\delta_i(t)$ is the angle of the i-th generator, in radian; $\omega_i(t)$ is the relative speed of the i-th generator, in rad/sec; Pmi is the mechanical input power, in p.u.; Pei (t) is the electrical power, in p.u.; ω_0 is the synchronous machine speed, in rad/sec, $\omega_0 = 2\pi f_0$; Di is the per unit damper constant; Hi is the inertia constant, in sec; E'qi(t) is the transient EMF in quadrature axis of the i-th generator, in p.u.; Efi(t) is the equivalent EMF in the excitation coil, in p.u.; Tdoi is the direct axis transient open circuit time constant, in second; Eqi is the EMF in quadrature axis of the i-th generator, in p.u.; Vti is the generator terminal voltage, in p.u.; xdi is the direct axis reactance of the ith generator, in p.u.; $x'di$ is the direct axis transient reactance of the i-th generator, in p.u.; Idi is the direct axis current, in p.u.; Iqi is the quadrature axis current, in p.u.; kci is the gain of the excitation amplifier, in p.u.; ufi(t) is the input of the SCR amplifier of the i-th generator; xadi is the mutual reactance between the excitation coil and the stator coil of the i-th generator; Yij=Gij+jBij is the i-th row and j-th column element of nodal admittance matrix, in p.u.; Qei is the reactive power, in p.u.; Ifi is the excitation current; $f_{i1}(t)$ and $f_{i2}(t)$ are highly nonlinear functions.

The classical third-order single-axis dynamic generator model used in this paper is referred in [14].

4.2 T-S fuzzy model of power system

Bae et al. in [15] presented a method of constructing the T-S fuzzy model using the sum of a product of linearly independent functions. The T-S fuzzy model of the power system adopted is constructed according to the improved Bae method [15]. From (15) and (16), we can find that $f_{i_1}(t)$ and$f_{i_2}(t)$ are dependent on the operating conditions but bounded with a certain operating region. The following bounds of $f_{i_1}(t)$ and$f_{i_2}(t)$ are considered:

$$-3.526 \leq f_{1_1} \leq -0.259,$$

$$0.266 \leq f_{1_2} \leq 3.794,$$
$$-2.832 \leq f_{2_1} \leq -0.233,$$
$$0.241 \leq f_{2_2} \leq 3.670.$$

According to [14], the nonlinear state equation (10) is expressed by

$$\dot{x_i}(t) = [F_{i_0} + \sum_{j=0}^{2} f_{i_j}(z(t))F_{i_j}]\eta_i(t), \tag{17}$$

where

$$x_i(t) = [\Delta V_{ti}(t), \Delta\omega_i(t), \Delta P_{ei}(t)]^T, \tag{18}$$

$$\eta_i(t) = [\Delta V_{ti}(t), \Delta\omega_i(t), \Delta P_{ei}(t), v_{fi}(t)]^T, \tag{19}$$

$$F_{i_0} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{D_i}{2H_i} & -\frac{\omega_0}{2H_i} & 0 \\ 0 & 0 & -\frac{1}{T'_d oi} & \frac{1}{T'_d oi} \end{pmatrix}, F_{i_1} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, F_{i_2} = \begin{pmatrix} 0 & 0 & -\frac{1}{T'_{d0i}} & \frac{1}{T'_{d0i}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{20}$$

As the number of linearly independent functions is 2 and for each function $f_{i_1}(t)$ and$f_{i_2}(t)$ two triangular fuzzy sets are assigned, 4 fuzzy rules are formulated. The T-S fuzzy model of the nonlinear system (10) is such that

$$\dot{x_i}(t) = \sum_{j=1}^{4} h_{i_j}(z(t))(A_{i_j}x(t) + B_{i_j}u(t)), \tag{21}$$

where
$h_{i_1} = M_{i_{10}}M_{i_{20}}$,$h_{i_2} = M_{i_{10}}M_{i_{21}}$, $h_{i_3} = M_{i_{11}}M_{i_{20}}$,$h_{i_4} = M_{i_{11}}M_{i_{21}}$,
$M_{i_{j0}}(z(t)) = \frac{(f_{i_{j1}} - f_{i_j}(z(t))}{(f_{i_{j1}} - f_{i_{j0}})}$,$M_{i_{j1}}(z(t)) = \frac{(f_{i_j}(z(t)) - f_{i_{j0}})}{(f_{i_{j1}} - f_{i_{j0}})}$,
$[A_{i_1}, B_{i_1}] = F_{i_0} + f_{i_{10}}F_{i_1} + f_{i_{20}}F_{i_2}$,$[A_{i_2}, B_{i_2}] = F_{i_0} + f_{i_{10}}F_{i_1} + f_{i_{21}}F_{i_2}$,
$[A_{i_3}, B_{i_3}] = F_{i_0} + f_{i_{11}}F_{i_1} + f_{i_{20}}F_{i_2}$,$[A_{i_2}, B_{i_2}] = F_{i_0} + f_{i_{11}}F_{i_1} + f_{i_{21}}F_{i_2}$.

5 Simulation Results

To evaluate the above control scheme for transient stability enhancement and voltage regulation, the example of two-machine three-bus power system is represented in Figure 2. The generator and the transmission line parameters are listed in Table 1 [14].

The performance of the proposed controller is tested under the following temporary fault sequence:
▷ Stage 1: The system is in a pre-fault steady state.
▷ Stage 2: A fault occurs at t=1s.
▷ Stage 3: The fault is removed by opening the breakers of the faulted line at t=1.15s.
▷ Stage 4: A mechanical input power of the generator1 has a 30% step increase at t = 2s.
▷ Stage 5: The system is in a post-fault state.

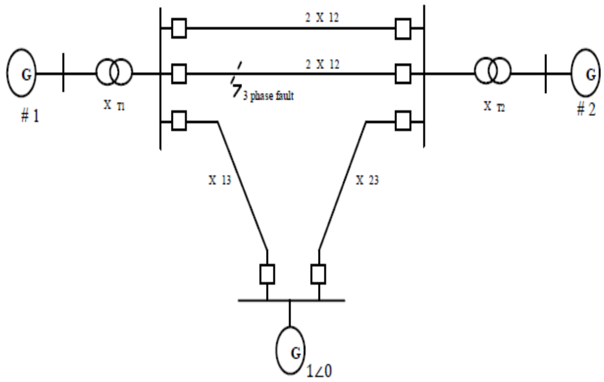

Figure 1: Two-machine infinite bus power system.

Generator	1	2
$x_d(p.u.)$	1.863	2.36
$x'_d(p.u.)$	0.257	0.319
$x_T(p.u.)$	0.129	0.11
$x_{ad}(p.u.)$	1.712	1.7126
$T'_{do}(sec)$	6.9	7.96
$H(s)$	4.0	5.1
$D(p.u.)$	5.0	3
$k_c(p.u.)$	1.0	1.0
$x_{12}(p.u.)$	0.55	
$x_{13}(p.u.)$	0.53	
$x_{23}(p.u.)$	0.6	
$\omega_0(rad/d)$	314.159	

Table 1: System parameters.

The following cases are considered.

• Case 1: Different sets of operating points: Two different operating points are considered:

Operating point 1:

$$\delta_{10} = 46.00°; Pm_{10} = 0.87p.u., V_{t10} = 1.0p.u.$$
$$\delta_{20} = 44.69°; Pm_{20} = 0.86p.u., V_{t20} = 1.0p.u.$$

Operating point 2:

$$\delta_{10} = 34.89°; Pm_{10} = 0.65p.u., V_{t10} = 1.02p.u.$$
$$\delta_{20} = 35.75°; Pm_{20} = 0.61p.u., V_{t20} = 1.09p.u.$$

The fault location is $\lambda = 0.02$. The corresponding closed loop system responses are shown in Figure 2 and Figure 3, respectively.

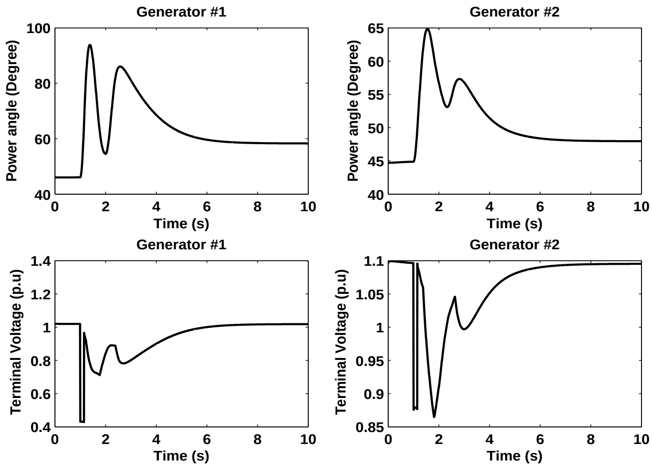

Figure 2: Power system responses for Case 1, operating point 1.

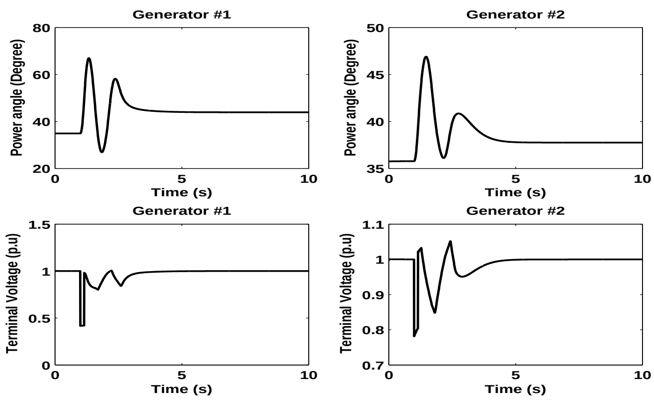

Figure 3: Power system responses for Case 1, operating point 2.

- Case 2: Fault location.

To test the ability of the proposed controller to achieve the proposed control task, two different fault locations are proposed $\lambda = 0.01$ and $\lambda = 0.5$. The operating point considered is

$$\delta_{10} = 18.51°; Pm_{10} = 0.3p.u., V_{t10} = 0.95p.u.$$
$$\delta_{20} = 23.68°; Pm_{20} = 0.4p.u., V_{t20} = 0.95p.u.$$

The corresponding closed loop system responses are shown in Figure 4 and Figure 5, respectively.

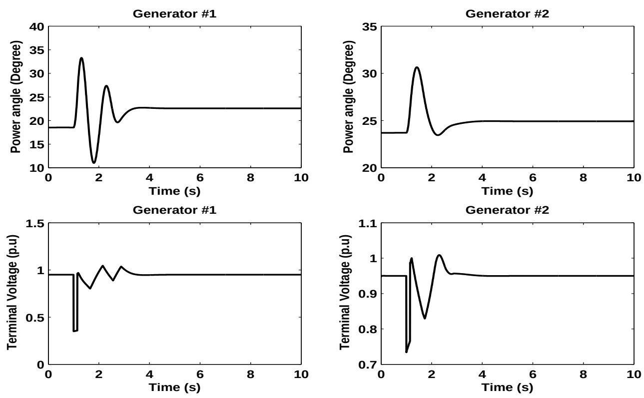

Figure 4: Power system responses for Case 2, fault location $\lambda = 0.01$.

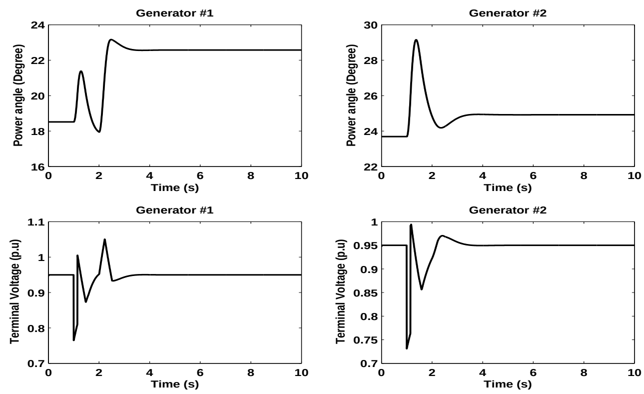

Figure 5: Power system responses for Case 2, fault location $\lambda = 0.5$.

Figures 2-5 show the system performances when subjected to different faults. It can be concluded from the simulation results that the proposed optimal voltage controller exhibits good transient performance: the oscillations are damped out effectively; the terminals voltages of generators are well regulated to their pre-fault values regardless of the operating points, change in the mechanical input power and fault locations.

6 Conclusion

In this paper, the design of optimal nonlinear state feedback voltage regulator for power systems based on the Takagi-Sugeno fuzzy model and parallel distributed compensation (PDC) scheme was presented. The proposed methodology reformulates the stability as convex optimization problems with linear matrix inequality (LMI). To demonstrate the effectiveness of the proposed controller, a two-machine three-bus power system has been considered. Simulation results show that both transient stability and voltage quality can be improved effectively regardless of the system operating conditions.

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