Nonlinear Dynamics and Systems Theory, **19** (1) (2019) 97–110

Existence of Renormalized Solutions for Some Strongly Parabolic Problems in Musielak-Orlicz-Sobolev Spaces

A. Talha *, A. Benkirane and M.S.B. Elemine Vall

Laboratory LAMA, Faculty of Sciences Dhar El Mahraz
University Sidi Mohamed Ben Abdellah, P.O. Box 1796, Atlas Fez, Morocco

Received: December 19, 2016; Revised: December 13, 2018

Abstract: In this work, we prove an existence result of renormalized solutions in Musielak-Orlicz-Sobolev spaces for a class of nonlinear parabolic equations with two lower order terms and L^1-data.

Keywords: *parabolic problems, Musielak-Orlicz space, renormalized solutions.*

Mathematics Subject Classification (2010): 46E35, 35K15, 35K20, 35K60.

1 Introduction

We consider the following nonlinear parabolic problem:

$$(\mathcal{P}) \begin{cases} \frac{\partial u}{\partial t} - \operatorname{div}\big(a(x,t,u,\nabla u) + \Phi(u)\big) + g(x,t,u,\nabla u) = f & \text{in } Q, \\ u = 0 \quad \text{on } \partial Q = \partial\Omega \times [0,T], \\ u(x,0) = u_0 \quad \text{on} \quad \Omega, \end{cases}$$

where $A(u) = -\operatorname{div}(a(x,t,u,\nabla u))$ is an operator of Leray-Lions type, the lower order term $\Phi \in C^0(\mathbb{R}, \mathbb{R}^N)$, g is a nonlinearity term which satisfies the growth and the sign condition and the data f belong to $L^1(Q)$. Under these assumptions the term $\operatorname{div}(\Phi(u))$ may not exist in the distributions sense, since the function $\Phi(u)$ does not belong to $(L^1_{\text{loc}}(Q))^N$.

In the setting of classical Sobolev spaces, the existence of a weak solution for the problem $(\mathcal{P})$ has been proved in [10] in the case of $\Phi \equiv g \equiv 0$. It is well known that this weak solution is not unique in general (see [16] for a counter-example in the stationary case).

* Corresponding author: mailto:talha.abdous@gmail.com

