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Mathematical Model of C_d for Circular Cylinder Using Two Passive Controls at Re = 5000

C. Imron [1*] C.J. Kumalasari [1] B. Widodo [2] and T.Y. Yuwono [2]

[1] *Department of Mathematics, Faculty of Mathematics, Computation and Data Science, Institute of Technology Sepuluh Nopember, Surabaya, Indonesia*
[2] *Department of Mechanical Engineering, Faculty of Industrial Technology, Institute of Technology Sepuluh Nopember, Surabaya, Indonesia*

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Abstract: This study focuses on two passive controls. Passive control is the addition of a small object to an object to reduce the drag force of the object. In this case, two passive controls are placed in front of and in the rear of the main object. The distance between the main object and the two passive controls varies and the Reynolds number used is 5000. The main object is a circular cylinder, and its passive control in front is a cylinder of type-I at the distance S / D = 0.6, 1.2 ; 1.8; 2.4; 3.0 and in the rear is an elliptical or circular cylinder at the distance T / D = 0,6; 0,9; 1,2; 1,5; 1,8 and 2,1. In this study, we want to find an effective distance of the main object to two passive controls so that the drag coefficient of the main object is minimal compared to that with non-passive control or with one passive control in front. In addition, a mathematical model of the drag coefficient of circular cylinders with two passive controls at Re = 5000 will be obtained.

Keywords: *passive control; drag coefficient; cylinder.*

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* Corresponding author: `mailto:imron-its@matematika.its.ac.id`

1 Introduction

Today, so many people are racing to create new technologies. Technological advancement is growing rapidly. Technology is actually a way and effort to improve the quality of human life [1]. New technologies can be created by conducting ongoing research, where the new technology is expected to change the behavior of users of these new technologies. Research related to fluid flow can be done by experiment or simulation. Study of the flow of fluids through objects with the aim of reducing the drag force of most objects is a paramount concern of the researchers.

Some researchers used one passive control placed in front of various shapes, such as cylindrical cylinders, type-I cylinders, type-D cylinders etc. Circular cylinders, elliptical cylinders or other shapes are commonly used objects for designing industrial chimneys, offshore and flyover structures and others. In this case, the design process should allow for the geometrical shape of the object because it affects the value of the drag coeffcient, so that for different geometric shapes the drag coeffcient values are also different. At the interaction between the fluid flow and the object the resulting fluid flow across a single object or multiple grouped objects will produce different flow characteristics.

In this study, we consider a boundary layer because it is seen that the liquid that flows through the surface of the object comes with the flow of particles around it. Basically, the boundary layer is an increase in shear stress which will affect the flow velocity in each layer [14]. The surface of the object will move slowly due to the friction force, so that the particle flow velocity around the object will be zero. While the other particles will interact,the velocity of the flow away from the object will be faster. This is due to increased shear stress.

There are some studies that use boundary layer concept, and the concept of the boundary layer can help to find the answer to the effect of shear stress having a very important role in flow characteristics around the object [2]. The research, among others, has been conducted on the flow of fluids through an object, such as a single cylindrical circular object [3], or a modified cylinder such as a cylinder of type-I or a cylinder of type-D [4,5] and a study has been conducted on a fluid stream through more than one object, i.e. fluid flow through more than one cylinder of various sizes and configurations, fluid flow through a circular cylinder with tandem configuration [6–9] and eliptical cylinders with their side configurations [10,11].

The existence of a drag force occurs when an object is bypassed by a fluid. In this case, the drag force is influenced by several parameters, one of which is the drag coefficient. One way to reduce the drag force on the objects bypassed is to add a smaller object in front of the main object called the passive control. The addition of passive control is carried out to reduce the coefficient by 48% [6], also one can find a mathematical model for a circular cylinder with two passive controls with the Reynolds number 5000. The cylinder of type-I is a circular cylinder obtained by cutting the left and right ends at a certain angle, so that the cylinder is shaped like I. The best cutting edge is 53^o, this is because the wake occured is wider than that at the other angle, forming also a wider and more annoying strong flow on the object wall.

In this study, we will get a mathematical model for a circular cylinder with two passive controls with the Reynolds number when these two passive controls effectively decrease the drag coefficient. The Reynolds number used is Re = 5000. Two passive controls will be used, the passive control in front is the cylinder of type-I and the passive control of type-I is placed perpendicular to the flow, while the passive control in the rear is landscape. The distance between the passive control in front and the circular cylinder is varying, as well as the distance between the passive control in the rear and the circular cylinder.

2 Numerical Method

The previously described problem can be solved by using the unstable incompressible fluid equation and the Navier-Stokes equation:

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot \mathbf{v}\mathbf{v} = -\nabla P + \frac{1}{\mathrm{Re}}\nabla^2 \mathbf{v}, \tag{1}$$

$$\nabla \cdot \mathbf{v} = 0, \tag{2}$$

where Re is the Reynolds number, $\mathbf{v}$ is the velocity, and P is the pressure. The Navier-Stokes equation can be solved by using SIMPLE algorithms and numerical methods. The first thing to do is to give the initial value for each variable. By ignoring the pressure components, we will find the velocity component of the momentum equation, so equation (1) becomes

$$\frac{\partial \mathbf{v}}{\partial t} = -\nabla \cdot \mathbf{v}\mathbf{v} + \frac{1}{\mathrm{Re}}\nabla^2 \mathbf{v} \tag{3}$$

by using the finite difference method, we have

$$(f_x)_i = \frac{2f_{i+1} + 3f_i - 6f_{i-1} + f_{i-2}}{6\,\mathrm{dx}} \quad \text{and} \quad (f_y)_j = \frac{2f_{j+1} + 3f_j - 6f_{j-1} + f_{j-2}}{6\,\mathrm{dx}},$$

$$(f_{xx})_i = \frac{f_{i+1} - 2f_i + f_{i-1}}{\mathrm{dx}^2} \quad \text{and} \quad (f_y y)_j = \frac{f_{j+1} - 2f_j + f_{j-1}}{\mathrm{dx}^2},$$

and afterwards

$$\frac{\partial \mathbf{v}}{\partial t} = \frac{\mathbf{v}^{**} - \mathbf{v}^{*}}{\Delta t} = -\nabla P \tag{4}$$

because of equation (2), then equation (4) becomes

$$\frac{\nabla \cdot \mathbf{v}^{*}}{\Delta t} = -\Delta P \tag{5}$$

by using *SOR* (*Successive Over Relaxation*)

$$(P_n)_{i,j} = (1-\epsilon)(P_{n-1})_{i,j} + \epsilon(P_n)_{i,j}. \tag{6}$$

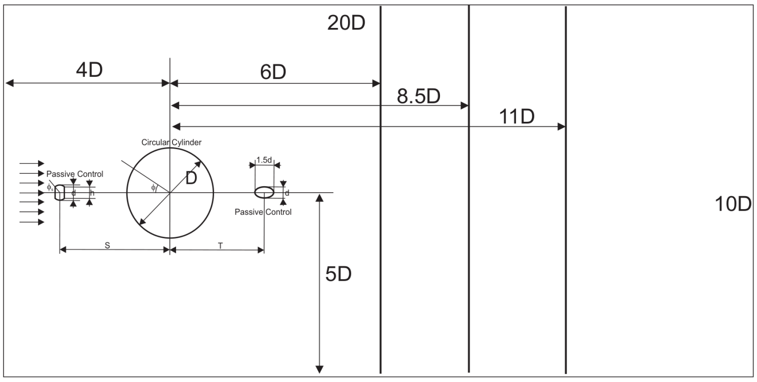

Figure 1: Design of the research system.

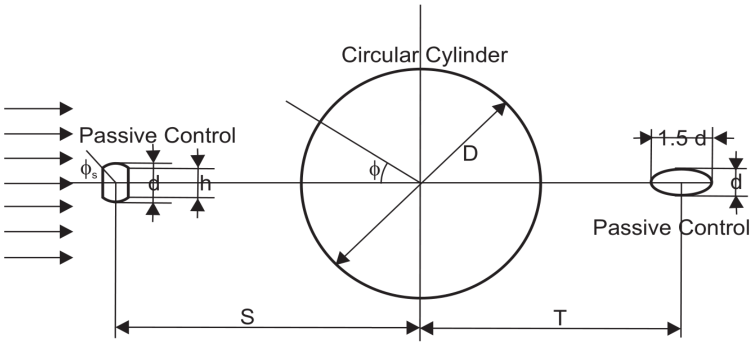

Figure 2: Schematic of two passive controls and a circular cylinder.

3 Main Result

Our research system is 10D 20D, where D is the diameter of the circular cylinder, placed at the distance of 4D from the front of the system and in the center of the system, as shown in Figure 1.

In this study, we used two passive controls. The first passive control is a cylinder of type-*I* placed in front of a circular cylinder at varying distance, i.e. S / D = 0.6,1,2,1,8,2,4 and 3.0. The second passive control are circular cylindrical and elliptical cylinders. The second passive control is placed in the rear of the circular cylinder at varying distance, i.e. T / D = 0.6,0,9,1,2,1,5,1,8 and 2.1 as shown in Figure 2.

3.1 Drag coefficient

The drag coeffcient of a single circular cylinder has been obtained by using the simulation program, the results are compared with experimental results and other simulation programs. We calculated that the drag coefficient of a single cylinder with Re = 100 is 1.356, while other researchers, with the same Reynolds number, have obtained: Zulhidayat has 1.4 and Five has 1.39 [12]. In this paper we will simulate a circular cylinder with two passive controls, and the Reynolds number used is 5000. The drag coefficient for a circular cylinder with Re = 5000 is 1.51.

S/D	0.6	1.2	1.8	2.4	3.0
C_D 5000	1.455	1.273	1.221	1.224	1.216

Table 1: Cd of a circular cylinder for Re=5000 with difference S/D.

Table 1 presents data on the drag coeffcient of a circular cylinder with a passive control, the cylinder of type-I, located at the front at varying distance. From the table it is clear that for the Reynold number Re = 5000, the best distance to get the minimum drag coeffcient is S/D = 1.8 or S/D = 3.0 with a drag coeffcient of 1.221 or 1.216. The value of the drag coeffcient is still smaller than the drag coeffcient without passive control.

C_{DO}	S/D				
T/D	0.6	1.2	1.8	2.4	3.0
0.6	1.116	1.012	0.973	0.992	0.987
0.9	1.205	1.043	1.015	1.007	1.008
1.2	1.169	1.014	0.977	0.990	0.986
1.5	1.412	1.277	0.916	1.265	1.245
1.8	1.557	1.284	1.225	1.222	1.195
2.1	1.401	1.384	1.220	1.209	1.191

Table 2: C_D of a circular cylinder for Re=5000 with difference S/D.

The drag coefficient of a circular cylinder with two passive controls at the front and in the rear. Passive control in front of the circular cylinder is the cylinder of type-I, while the passive control behind the circular cylinder is a small circular cylinder. The data on the drag coefficient with the Reynolds number Re = 5000 and the configuration as above, can be seen in Table 2. It appears that the passive control behind has a significant effect on the drag coeffcient, since the drag coeffcient is still smaller than that without passive control. The minimum drag coefficient of the configuration is 0.916, this occurs at S/D = 1.8 and T/D = 1.5.

3.2 Mathematical Model

In this case, the simulation result of the drag coefficient with two passive controls is interpolated to obtain the mathematical model. By using the bilinear interpolation approach one can make a mathematical model of the drag coefficient. Bilinear interpolation is the development of linear interpolation of two variables [13]. In this study we use the 2nd order bilinear interpolation. In this case, the variables used are (x, y) = (T / D, S / D). By taking the nine points of drag data that have been obtained from the simulation results in Table 2 we can get the interpolation formulation. Therefore, nine polynomial equations and nine unknown coefficients can be obtained. The polynomial interpolation function can be written as follows:

$$f(x,y) = a_{00} + a_{01}y + a_{02}y^2 + a_{10}x + a_{11}xy + a_{12}xy^2 + a_{20}x^2 + a_{21}x^2y + a_{22}x^2y^2. \quad (7)$$

Taking data from Table 3 and substituting (x, y)=(T/D, S/D) into f(x,y) we find the unknown coefficients. Therefore, we can obtain the mathematical model of the drag coefficient as follows:

$$\begin{aligned} E(x,y) &= 0.0275x^2y^2 - 0.0240x^2y + 0.0590x^2 - 0.1713xy^2 + 0.1792xy - 0.2958x \\ &\quad +5.5913y^2 - 0.848y + 1.5542. \end{aligned} \quad (8)$$

$f(T/D, S/D)$	S/D		
T/D	0.6	1.8	3.0
0.6	1.116	0.973	0.987
1.2	1.169	0.977	0.986
1.8	1.557	1.225	1.195

Table 3: Nine drag data results for a circular cylinder

The error in the above mathematical model is calculated using an absolute error as follows:

$$e(x,y) = |E(x,y) - f(x,y)|. \quad (9)$$

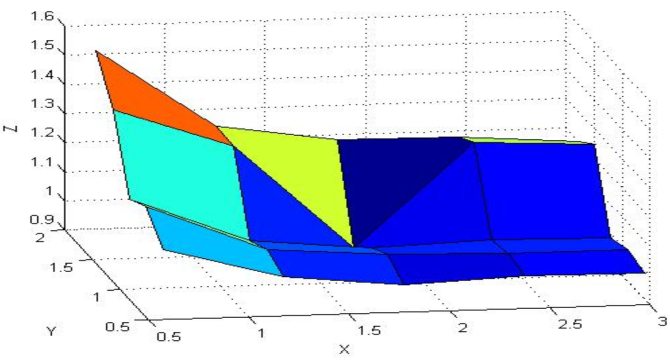

Figure 3: Graphic plot of Table 2 in Matlab.

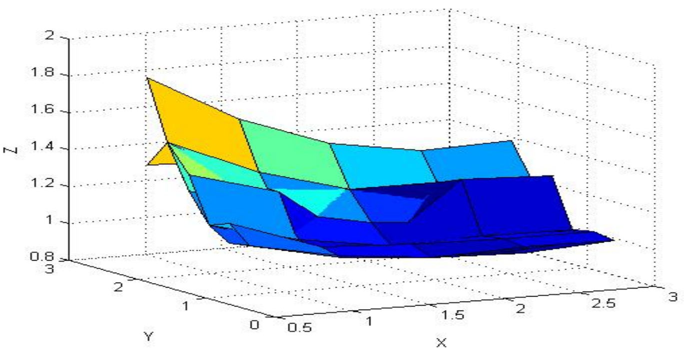

Figure 4: Comparison of the graphic plot of Table 2 and the graphic plot of Table 3.

If we simulate the original data in Table 2 as shown in Figure 3 and compare it with the simulation result by using bilinear interpolation shown in Figure 4 then it appears that the smallest absolute error is S / D = 0.6,1.8,3.0 , T / D = 0.6,1.2,1.8 with the value of Cd = 1.116, 1.169, 1.557, 0.973, 0.977, 1.225, 0.987, 0.986, 1.195 and also the obtained largest absolute error is in S / D =2.4 , T / D = 1.5 , with the value of Cd = 1.265. In other words, the error will not exceed the point of 0.2282.

3.3 Wake

In this study, the velocity data at the distance 6D, 8.5D and 11D from the center of the circular cylinder or at the distance 10D, 12.5D and 15D from the front of the system, are shown in Figure 1 . In both passive controls with Re = 5000 there is a wake. Also, it can be seen for the drag coefficient of the main circular cylinder that there is a significant decrease. In this case there is a decrease in the drag coefficient which affects the magnitude of the average velocity behind the circular cylinder.

It appears that Table 4 shows that a decrease in flow velocity behind the circular cylinder correspons to the decrease in the drag coefficient. In addition, the flow velocity near the circular cylinder (i.e., 6D from the center of the circular cylinder) will increase as the distance moves farther away from the center of the circular cylinder and will return equally to the speed without passive control.

Re	**Single**	**1 PC**	**%**	**2 PC**	**%**
5000	1.51	1.216	19.47	0.916	39.34

Table 4: C_D for $R_e = 5000$.

4 Conclusion

By using the bilinear interpolation approach one can make a mathematical model of the drag coefficient. Bilinear interpolation is the development of linear interpolation of two variables. In this study we use the 2nd order bilinear interpolation. Thus, a mathematical model can be formed for C_d of a circular cylinder using two passive controls at Re = 5000. The mathematical model can be written as follows:

$$\begin{aligned} E(x,y) &= 0.0275x^2y^2 - 0.0240x^2y + 0.0590x^2 - 0.1713xy^2 + 0.1792xy - 0.2958x \\ &\quad +5.5913y^2 - 0.848y + 1.5542. \end{aligned} \tag{10}$$

In addition, a reduction of the drag coefficient in a circular cylinder can be done by adding passive control. Passive control can be placed in front and/or behind. The drag coefficient can be reduced by up to 40% if using passive control in the form of the cylinder of type-I and an ellipse-shaped cylindrical back control rather than a passive control drag coefficient. This is reinforced by the decreasing flow velocity behind the circular cylinder.

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