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Existence of Solutions for a Biological Model Using Topological Degree Theory

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Abstract: Topological degree theory is a useful tool for studying systems of differential equations. In this work, a biological model is considered. Specifically, we prove the existence of positive T-periodic solutions of a system of delay differential equations for a model with feedback arising on circadian oscillations in the drosophila period gene protein.

Keywords: *differential equations with delay; periodic solutions; models with feedback; topological degree; drosophila; circadian cycle.*

Mathematics Subject Classification (2010): 34K13, 92B05.

1 Introduction

The study of cellular control has been developed in many papers on mathematical analysis to determine the existence of stable oscillations in mRNA regulatory processes, see [5] and to understand circadian cycles and, in particular, of the cellular machinery that produces them, see [7].

In all cases, search for conditions on the parameters of the proposed systems has been carried out with the purpose of determining conditions for the existence of stable cycles and the cycles when the system solution may be even chaotic.

Let us consider a model proposed by Goldbeter [3], who showed the variation on PER: the period of *messenger of Ribo-Nucleic Acid (mRNA)* in Drosophila (often called "fruit flies") related to circadian rhythms. Our model does not consider temperature variation as shown in [6]. Here, a nonautonomous version of the model is considered with the aim of proving the existence of periodic solutions by means of a powerful topological tool: the Leray-Schauder degree (see [1] and [2]). In the original model, the existence of a positive steady state can be shown, under appropriate conditions, by the use of the Brouwer degree. As we shall see, when the parameters are replaced by periodic functions, essentially the same conditions yield the existence of positive periodic solutions.

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