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Complete Symmetry and μ-Symmetry Analysis of the Kawahara-KdV Type Equation

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Abstract: The goal of this paper is complete analysis of the Kawahara-KdV type equation using the ordinary symmetry and μ-symmetry methods. In other words, the Lie symmetry, symmetry reduction, differential invariant and conservation laws for the Kawahara-KdV type equation are provided. And in the second part the μ-symmetry, order reduced equations, Lagrangian and μ-conservation laws for the Kawahara-KdV type equation are presented.

Keywords: *Lie symmetry; μ-symmetry; Kawahara-KdV type equation; symmetry reduction; differential invariant; conservation law; order reduced equations; Lagrangian; variational problem; μ-conservation law.*

Mathematics Subject Classification (2010): 58D19, 58J70, 70G65, 76M60.

1 Introduction

The symmetry method is a powerful tool of differential geometry for accurate analysis of a mathematical model as a description of a system in many areas of applied mathematics and physics. Dispersive wave equations arise in many areas when the third order derivative in the KdV (Korteweg de Vries) equation approaches zero. It is necessary to take account of the higher order effect of dispersion in order to balance the nonlinear effect.

The Kawahara-KdV equation, modified Kawahara-KdV equation and Kawahara-KdV type equation, respectively, are given as:

$$u_t + uu_x + u_{xxx} - \gamma_1 u_{xxxxx} = 0, \quad u_t + 3u^2u_x + u_{xxx} - \gamma_2 u_{xxxxx} = 0,$$
$$u_t + u_x + uu_x + u_{xxx} - \gamma u_{xxxxx} = 0, \tag{1}$$

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