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On Stability of a Second Order Integro-Differential Equation

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Abstract: There exists a well-developed stability theory for integro-differential equations of the first order and only a few results on integro-differential equations of the second order. The aim of this paper is to fill up this gap. Explicit tests for uniform exponential stability of linear scalar delay integro-differential equations of the second order

$$\ddot{x}(t) + \int_{g(t)}^{t} G(t,s)\dot{x}(s)ds + \int_{h(t)}^{t} H(t,s)x(s)ds = 0$$

are obtained.

Keywords: *exponential stability; second order delay integro-differential equations; a priory estimation; Bohl-Perron theorem.*

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1 Introduction

Beginning with the classical book of Volterra [1] integro-differential equations and, more generally, functional differential equations have many applications in biology, physics, mechanics (see, for example, [2, 4–7, 22, 26]). In particular, second order integro-differential equations appear in stability problems of viscoelastic shells [3]. There are many papers devoted to stability of the first order integro-differential equations [8–11, 18] and only few papers on stability for the second order equations [12–14]. Oscillation conditions for the first and the second order functional differential equations can be found in papers [15–17].

The aim of the present paper is to fill up this gap and obtain new explicit exponential stability conditions for the equation

$$\ddot{x}(t) + \int_{g(t)}^{t} G(t,s)\dot{x}(s)ds + \int_{h(t)}^{t} H(t,s)x(s)ds = 0. \tag{1}$$

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