Nonlinear Dynamics and Systems Theory, **18** (4) (2018) 342–358

Krasnoselskii's Theorem, Integral Equations, Open Mappings, and Non-Uniqueness

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Received: June 6, 2018; Revised: October 12, 2018

Abstract: We study Krasnoselskii's fixed point theorem on the sum of two operators restricted to the Banach space of continuous functions with the supremum norm. The work is based on "open mappings" in the sense that our mapping P maps a closed bounded convex set M into its interior M^o. We show that any fixed point of a mapping of the whole space must reside in M^o. This is very informative in case of non-uniqueness. We also extend a known transformation to hold for integral equations being the sum of a contraction and a compact map where the "forcing function" is the contraction. Several examples are given showing the construction of the unusually simple mapping sets.

Keywords: *Krasnoselskii's fixed points; open mappings; transformations; uniqueness.*

Mathematics Subject Classification (2010): 34A08, 34A12, 45D05, 45G05, 4H09.

1 Introduction

Much has been written about fixed point mappings which are either contractions or compact. But around 1954 Krasnoselskii studied a paper by Schauder on differential equations and concluded a variant of the idea that the inversion of a perturbed differential operator yields the sum of a contraction and a compact map. Embodied in that theorem are both Banach's contraction mapping principle and Schauder's second fixed point theorem. All three of these are conveniently found in the monograph by Smart [15]. Accordingly, Krasnoselskii offered the following fixed point theorem [15, p. 31].

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