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Solvability Criterion for Integro-Differential Equations with Degenerate Kernel in Banach Spaces

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Abstract: By means of the theory of generalized inversion of operators in Banach spaces, a solvability criterion and a general form of solutions for integro-differential equations with a degenerate kernel in Banach spaces have been established. The obtained results have been illustrated by examples.

Keywords: *integro-differential equation; degenerate kernel; Banach space; generalized invertible operator; general form of solutions.*

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1 Introduction

The investigation of the solvability of integro-differential equations is a problem the specific nature of which lies in the fact that the integro-differential operator has no inverse. Such equations in Euclidean spaces were considered in [1–4] and others.

Sufficient conditions for the existence and uniqueness of piecewise-continuous mild solutions of fractional integro-differential equations in a Banach space with non instantaneous impulses were obtained in [5]. In paper [6] V. Gupta and J. Dabas established the existence and uniqueness of solution for a class of impulsive fractional integro-differential equations with nonlocal boundary conditions.

In this paper, we propose a somewhat different approach to the study of integro-differential equations in Banach spaces. In its realization, the theory of generalized inversion of operators in Banach spaces is effectively used [7, 8].

The proposed approach can be used in the study of the phenomena of energy transfer and diffusion of neutrons, viscoelastic oscillations various systems and structures, in nuclear physics and the mathematical theory of biological populations (see [9–11]).

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