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Existence of Solution for Nonlinear Anisotropic Degenerated Elliptic Unilateral Problems

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Abstract: In this paper, we prove the existence of entropy solutions of anisotropic elliptic equations $Au + \sum_{i=1}^{N} g_i(x,u,\nabla u) = f$, where the operator Au is a Leray-Lions anisotropic operator from $W_0^{1,\overrightarrow{p}}(\Omega,\overrightarrow{\omega})$ into its dual $W^{-1,\overrightarrow{p}'}(\Omega,\overrightarrow{\omega^*})$. The critical growth condition on g_i is with respect to ∇u and there is no the growth condition with respect to u and no the sign condition. The right-hand side f belongs to $L^1(\Omega)$.

Keywords: *nonlinear elliptic equations; quasilinear degenerated unilateral problems; non-variational inequalities.*

Mathematics Subject Classification (2010): 35J60, 35J70, 35J87.

1 Introduction

Let Ω be a bounded open subset of $\mathbb{R}^N$ $(N \geq 2)$ with Lipschitz continuous boundary and let $Au = -\sum_{i=1}^{N} \partial_i a_i(x,u,\nabla u)$ be a degenerate anisotropic operator of Leray-Lions type defined in the weighted anisotropic Sobolev space $W^{1,\overrightarrow{p}}(\Omega,\overrightarrow{\omega})$, where $\overrightarrow{\omega} = (\omega_0,\omega_1,...,\omega_N)$ is a vector of weight functions defined on Ω and $\overrightarrow{p} = (p_0,...,p_N)$ is a vector of real number such that $p_i > 1$ for $i = 0,...,N$.

We consider the following nonlinear elliptic anisotropic problem

$$\begin{cases} -\sum_{i=1}^{N} \partial_i a_i(x,u,\nabla u) + \sum_{i=1}^{N} g_i(x,u,\nabla u) = f & \text{in} \quad \Omega, \\ u = 0, & \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

where $g_i(x,s,\xi)$ is a Carathéodory function satisfying only the following growth condition $|g_i(x,s,\xi)| \leq \gamma(x) + \rho(s)|\xi_i|^{p_i}$ and where the right-hand side f belongs to $L^1(\Omega)$. In the

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