



# Monotone Method for Finite Systems of Nonlinear Riemann-Liouville Fractional Integro-Differential Equations

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**Abstract:** In this paper we develop the monotone method for nonlinear finite  $N$ -systems of Riemann-Liouville integro-differential equations of order  $0 < q < 1$ . The iterative technique approximates maximal and minimal coupled quasisolutions to the nonlinear system using sequences of linear systems that are constructed via coupled lower and upper solutions of varying types. Preliminary existence and comparison theorems are presented and proven where appropriate. Finally, we present a numerical example.

**Keywords:** *monotone method; Riemann-Liouville fractional integro-differential equations; finite systems.*

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## 1 Introduction

Fractional differential equations have various applications in widespread fields of science, such as engineering [5], chemistry [14, 15], physics [1, 8], and others [9, 10]. Despite there being a number of existence theorems for nonlinear fractional differential equations, much as in the integer order case, this does not necessarily imply that calculating a solution explicitly will be routine, or even possible. Therefore, it may be necessary to employ an iterative technique to numerically approximate a needed solution. In this paper we

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