Nonlinear Dynamics and Systems Theory, **18** (2) (2018) 107–129

Entropy Solutions of Nonlinear $p(x)$-Parabolic Inequalities

Youssef Akdim *, Allalou Chakir, Nezha El gorch and Mounir Mekkour

Sidi Mohamed Ben Abdellah University, Laboratory LSI Poly-Disciplinary Faculty of Taza P.O. Box 1223, Taza Gare, Morocco

Received: July 24, 2017; Revised: March 29, 2018

Abstract: In this paper we prove the existence of entropy solutions for weighted $p(x)$-parabolic problem associated with the equation:

$$\frac{\partial u}{\partial t} + Au = g(u)\omega(x)\Big|\nabla u\Big|^{p(x)} + f \quad \text{in} \quad \Omega \times (0,T),$$

where the operator $Au = -\text{div}\Big(\omega(x)\Big|\nabla u\Big|^{p(x)-2}\nabla u\Big)$ and on the right-hand side f belongs to $L^1(\Omega \times (0,T))$ and $\omega(x)$ is a weight function.

Keywords: *parabolic problems; entropy solutions; Sobolev space with variable exponent; penalized equations.*

Mathematics Subject Classification (2010): 47A15, 46A32, 47D20.

1 Introduction

Let Ω be an open bounded subset of $\mathbb{R}^N$, $N \geq 2$, T be a positive real number and $Q = \Omega \times (0,T)$, while the variable exponent $p: \quad \bar{\Omega} \to (1,\infty)$ is a continuous function, the data $f \in L^1(Q)$ and $u_0 \in L^1(\Omega)$. The objective of this paper is to study the existence of an entropy solution for the obstacle parabolic problems of the type:

$$\begin{cases} u \geq \psi, & \text{a.e. in } \Omega \times (0,T), \\ \frac{\partial u}{\partial t} - \text{div}\Big(\omega(x)\Big|\nabla u\Big|^{p(x)-2}\nabla u\Big) = \omega(x)g(u)\Big|\nabla u\Big|^{p(x)} + f, & \text{in } \Omega \times (0,T), \\ u(x,0) = u_0, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega \times (0,T). \end{cases} \tag{1}$$

* Corresponding author: mailto:akdimyoussef@yahoo.fr

© 2018 InforMath Publishing Group/1562-8353 (print)/1813-7385 (online)/http://e-ndst.kiev.ua