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Some Gronwall Lemmas Using Picard Operator Theory: Application to Dynamical Systems

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Abstract: In this paper we derive optimal explicit bounds for the solutions to integral inequalities. We rewrite the inequalities in terms of integral operators and we get the bound as a fixed point of the corresponding operator. As application, we study the stability of ceratin dynamical systems.

Keywords: *Gronwall lemma; dynamical system; fixed point; Picard operators.*

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1 Introduction

Integral inequalities are a necessary tool in the study of various classes of equations. In 1919, Gronwall [10] introduced the famous Gronwall inequality in the study of the solutions of differential equations. Since then, many contributions have been made (see [1]-[3]). The applications of integral inequalities were developed in a remarkable way in the study of the existence, the uniqueness, the comparison, the stability and continuous dependence of the solution in respect to data. In the last few years, a series of generalizations of these inequalities appeared. The problem of stability can be solved by Lyapunov techniques for differential equations (see [12]- [14]), or in terms of nonlinear integral inequalities. These inequalities can be used in the analysis of various problems in the theory of nonlinear differential equations and control systems (see [3] and references therein). There is an extensive literature on the inequalities, for example, the Barbalats lemma is an integral inequality used in applied nonlinear control. The second Lyapunov method has long played an important role in the history of stability theory, and it will with no doubt continue to serve as an indispensable tool in future research

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