



Weak Solutions to Implicit Differential Equations Involving the Hilfer Fractional Derivative

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Abstract: In this paper, the authors present some existence results for weak solutions to some functional implicit fractional differential equations of Hilfer type, by applying Mönch's fixed point theorem associated with the technique of measure of weak noncompactness.

Keywords: *functional differential equation; left-sided mixed Pettis Riemann-Liouville integral of fractional order; Hilfer fractional derivative; implicit; weak solution; fixed point.*

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1 Introduction

Fractional differential equations have recently been applied in various areas of engineering, mathematics, physics, bio-engineering, and other applied sciences [15, 24]. For some fundamental results in the theory of fractional calculus and fractional differential equations, we refer the reader to the monographs of Abbas *et al.* [1–3], Samko *et al.* [23], Kilbas *et al.* [18], and Zhou [27].

The notion of a measure of weak noncompactness was introduced by De Blasi [13]. The strong measure of noncompactness was developed first by Banaś and Goebel [7] and subsequently developed and used in many papers; see, for example, Akhmerov *et al.* [5], Alvàrez [6], Benchohra *et al.* [11], Guo *et al.* [14], and the references therein. In [11, 21], the authors considered some existence results by applying the techniques of the measure

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