



## Refinements of Some Pachpatte and Bihari Inequalities on Time Scales

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**Abstract:** In this study, some generalizations and refinements of some inequalities of Pachpatte and Bellman-Bihari types are established on arbitrary time domains using the time scale theory. The obtained results unify continuous and discrete inequalities and extend some results known in the literature. The paper ends up with two illustrative examples to highlight the utility of our results.

**Keywords:** *dynamic equations; time scale, Gronwall-Bellman inequality; Bellman-Bihari inequality.*

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### 1 Introduction

The Gronwall-Bellman and Bellman-Bihari integral inequalities play important roles in the study of qualitative and quantitative properties of differential equations [1–6]. Similarly, discrete Gronwall and Bihari inequalities have been developed for the analysis of difference equations [7]. New classes of differential and integral equations have been studied using Gronwall-Bellman-Pachpatte inequalities [5, 8, 9]. Recently, the time scale theory, which was introduced in [10], gives a promising direction that unifies continuous

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