Nonlinear Dynamics and Systems Theory, 17 (4) (2017) 347–356

Output Tracking of Some Class Non-minimum Phase Nonlinear Uncertain Systems

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Received: September 28, 2016; Revised: October 30, 2017

Abstract: In this paper, we present the output tracking for a class of non-minimum phase nonlinear uncertain systems. To achieve the output tracking, we will apply the modified steepest descent control. To apply the modified steepest descent control, the output of the system will be redefined so that the system will become minimum phase with respect to a new output.

Keywords: *relative degree of system; minimum phase system; non-minimum phase system; modified steepest descent control.*

Mathematics Subject Classification (2010): 93C10, 93D21.

1 Introduction

In the output tracking theory, the input-output linearization is one of the most available methods [1]. Output tracking problems for nonlinear non-minimum phase systems are a rather difficult issue in control theory. Most of researchers restrict their research to some special nonlinear classes only. The stable inversion proposed in [2], [3] is an iterative solution to the tracking problem with the unstable zero dynamics. This method requires the system to have well defined relative degree and hyperbolic dynamics, i.e. no eigenvalues on the imaginary axis. In [4], control design procedure for the output tracking was proposed. The design procedure consists of two steps. At the first step, the

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standard input-output linearization is applied. At the second step, we group an output with the internal dynamics as one subsystem, which is usually nonlinear, and the rest of the output as the other subsystem that is linear, the nonlinear subsystem is linearized about its equilibrium. In [5], the asymptotic output tracking which is a class of causal non-minimum phase uncertain nonlinear systems is achieved by using higher order sliding modes (HOSM) without reduction of the input-output dynamics order. Results on stabilization of non-minimum phase system in the output feedback form have been presented in [6], [7], [8]. The main idea in [6], [7], [8] is output reconstruction such that the system becomes minimum phase with respect to a new output. Results on output tracking of some class non-minimum phase nonlinear system have been presented in [9], [10]. In [9], the design of the input control is based on the exact linearization.

In this paper, we will modify the steepest descent control for output tracking of a class of non-minimum phase nonlinear uncertain systems, with relative degree being $n-1$, n is the dimension of the system. The modification is the addition of an artificial input of the steepest descent control. The design of descent control can not be initiated from the output causing the system to be non-minimum phase. In this paper, to solve the problem, we transform the system into a normal form which is minimum phase with respect to a virtual output, which is a linear combination of state variables.

2 Problem Statement

Consider nonlinear uncertain system

$$\dot{x} = \mathbf{A}x + \phi(y) + \theta\psi(y) + bu, \; x(t) \in \mathbf{R}^n, \; u(t) \in \mathbf{R}, \tag{1}$$

$$y = x_1, \tag{2}$$

in which $\phi(x)$ is smooth vector field in $\mathbf{R}^n$, with $\phi(0) = 0$, $\phi(y) = [\phi_1(y), \phi_2(y), \ldots, \phi_n(y)]^T$, $\psi(0) = 0$, $\theta\psi(y) = [\theta_1(t)\psi_1(y), \theta_2(t)\psi_2(y), \ldots, \theta_n(t)\psi_n(y)]$, $b = [0, \ldots, 0, b_{n-1}, b_n]^T$,

$$b_{n-1} \neq 0, \; b_{n-1} = -b_n \text{ and } \mathbf{A} = \begin{pmatrix} 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \\ 0 & 0 & \ldots & 0 \end{pmatrix}.$$

The relative degree of the system (1)-(2) is $n-1$.

The system (1)-(2) can be transformed to

$$\dot{z}_1 = z_2 + \theta_1(t)\psi(x_1), \tag{3}$$

$$\dot{z}_k = z_{k+1} + \varphi_{k-1}(t, x_1, \ldots, x_{k-1}), \quad k = 2, \ldots, n-2, \tag{4}$$

$$\dot{z}_{n-1} = a(z\ ,\ \eta) + b(z\ ,\ \eta)u + \varphi(t, x_1, \ldots, x_{n-2}), \tag{5}$$

$$\begin{aligned} \dot{\eta} &= \dot{x}_1 + \dot{x}_2 + \ldots + \dot{x}_n \\ &= \eta - z_1 + \phi_1(y) + \ldots + \phi_n(y) + \theta_1(t)\psi_1(y) + \ldots + \theta_n(t)\psi_n(y), \\ y &= z_1, \end{aligned}$$

with the internal dynamics

$$\dot{\eta} = \eta - z_1 + \phi_1(y) + \ldots + \phi_n(y) + \theta_1(t)\psi_1(y) + \cdots + \theta_n(t)\psi_n(y). \tag{6}$$

Then the zero dynamics of the system (1)-(2) is

$$\dot{\eta} = \eta.$$

Thus the system (1)-(2) is non-minimum phase.

Our objective is to make the output system (2) track the desired output. To make the system (1)-(2) track the desired output, we will use the dynamic feedback control. The design of the dynamic control is based on the modification of the steepest descent control. By "Trajectory Following Method" [11], the steepest descent control is determined from the differential equation $\dot{u} = -\frac{\partial F}{\partial u}$, where F is a descent function which has a variable as the solution of internal dynamics system. So, the modification of the steepest descent control can not be initiated from the output causing the system to be non-minimum phase. Therefore, the output of the system will be redefined so that the system will become minimum phase with respect to a new output.

3 Main Results

We consider system (1). Consider now a new output $\mu = t_1 x$, with $t_1 = (\alpha \ \ 1 \ \ 1 \ \ \ldots \ \ 1)$. The relative degree of system (1) with respect to μ is $n-1$. The system (1) with respect to μ, can be transformed to

$$\begin{aligned}
\dot{z}_1 &= z_2 + c\theta(t)\psi(x_1), && (7)\\
\dot{z}_k &= z_{k+1} + \omega_{i-1}(t, x_1, \ldots, x_{i-1}), \qquad k = 2, \ldots, n-2, && (8)\\
\dot{z}_{n-1} &= a(z\ ,\ \eta)\ +\ b(z\ ,\ \eta)u + \omega(t, x_1, \ldots, x_{n-2}), && (9)\\
\dot{\eta} &= \dot{x}_1 + \dot{x}_2 + \ldots + \dot{x}_n \\
&= \eta - x_1 + \phi_1(x_1) + \ldots + \phi_n(x_1) + \theta_1(t)\psi_1(x_1) + \ldots + \theta_n(t)\psi_n(x_1),\\
y &= \mu = z_1.
\end{aligned}$$

Furthermore

$$\eta\dot{\eta} = \eta(\eta - x_1 + \phi_1(x_1) + \ldots + \phi_n(x_1) + \theta_1(t)\psi(x_1) + \cdots + \theta_n(t)\psi_n(x_1). \tag{10}$$

Assumption 3.1 $\psi_i(x_1) \leq |x_1|$, $\forall x_1,\ i = 1, 2, \ldots, n$.

Case 1 : if $\phi_1(x_1) + \phi_2(x_1) + \cdots + \phi_n(x_1) = 0$.
Then

$$\begin{aligned}
\eta\dot{\eta} &= \eta^2 - \eta x_1 + \eta\theta_1(t)\psi_1(x_1) + \cdots + \eta\theta_n(t)\psi_n(x_1)\\
&\leq \eta^2 - \eta x_1 + |\eta||x_1| \left(|\theta_1(t)| + |\theta_2(t)| + \ldots + |\theta_n(t)|\right)\\
&= \eta^2 - \eta\left(\frac{z_1 - \eta}{\alpha - 1}\right) + |\eta|\left|\frac{z_1 - \eta}{\alpha - 1}\right| \left(|\theta_1(t)| + |\theta_2(t)| + \ldots + |\theta_n(t)|\right).
\end{aligned}$$

Then if $z_1 = 0$ and $0 < \alpha < 1$, we have

$$\eta\dot{\eta} \leq \eta^2 \left(\frac{-\alpha + (|\theta_1(t)| + |\theta_2(t)| + \ldots + |\theta_n(t)|)}{|\alpha - 1|}\right). \tag{11}$$

If $|\theta_1(t)| + |\theta_2(t)| + \ldots + |\theta_n(t)| \ < \ \alpha$, then $\eta\dot{\eta} \ < \ 0$. Therefore, the zero dynamics (1) with respect to output μ is asymptotically stable. Thus the system (1) with respect to output μ is minimum phase.

Case 2 : if $\phi_1(x_1) + \phi_2(x_1) + \cdots + \phi_n(x_1) = h(x_1) \neq 0$.
We have

$$\begin{aligned}\eta\dot{\eta} &= \eta(\eta - x_1 + h(x_1) + \theta_1(t)\psi(x_1) + \cdots + \theta_n(t)\psi_n(x_1)) \\ &\leq \eta^2 - \eta x_1 + \eta h(x_1) + |\eta||x_1|\left(|\theta_1(t)| + |\theta_2(t)| + \ldots + |\theta_n(t)|\right) \\ &= \eta^2 - \eta\left(\frac{z_1-\eta}{\alpha-1}\right) + \eta h\left(\frac{z_1-\eta}{\alpha-1}\right) + |\eta|\left|\frac{z_1-\eta}{\alpha-1}\right|\left(|\theta_1(t)| + \ldots + |\theta_n(t)|\right).\end{aligned}$$

If $z_1 = 0$, $\forall t$ and $0 < \alpha < 1$, then

$$\eta\dot{\eta} \leq \eta^2\left(\frac{-\alpha + (|\theta_1(t)| + |\theta_2(t)| + \ldots + |\theta_n(t)|)}{|\alpha-1|}\right) + \eta h\left(\frac{-\eta}{\alpha-1}\right). \tag{12}$$

Assumption 3.2 We consider system (1). Choose $\phi_1(x_1)$, $\phi_2(x_1)$, …, $\phi_n(x_1)$ so that

$$\eta h\left(\frac{-\eta}{\alpha-1}\right) < 0.$$

If $|\theta_1(t)| + |\theta_2(t)| + \ldots + |\theta_n(t)| \leq \alpha$ and by Assumption 3.2, we have $\eta\dot{\eta} < 0$. Therefore the system (1) with respect to output μ is minimum phase.

Lemma 3.1 *Consider system (1). Then there exists a linear combination of the state variables $\mu = \alpha x_1 + x_2 + x_3 + \ldots + x_n$ such that the relative degree of the system (1) with respect to output μ is $n-1$. Furthermore due to Assumption 3.1 we obtain*
(i) If $\phi(x_1) + \cdots + \phi_n(x_1) = 0$, the system (1) with respect to output μ is minimum phase, with $\mid \theta_1(t) \mid + \mid \theta_2(t) \mid + \ldots + \mid \theta_n(t) \mid < \alpha$, $0 < \alpha < 1$.
(ii) If $\phi(x_1) + \cdots + \phi_n(x_1) \neq 0$ and by Assumption 3.2, the system (1) with respect to output μ is minimum phase, with $0 < \alpha < 1$ and $\mid \theta_1(t) \mid + \ldots + \mid \theta_n(t) \mid \leq \alpha$.

Let μ_d be the desired output of the new output.

Assumption 3.3 Let $x_i = x_{id}$, $i = 1, 2, \ldots, n-2$.

Based on Assumption 3.3, we have x_{2d}, x_{3d}, …, $x_{(n-1)d}$, respectively. Then $\dot{x}_n = f(x_1, x_{n-1}, x_n)$ can be solved by substituting $x_{n-1} = x_{(n-1)d}$. Thus $x_n = x_{nd}$. Furthermore the definition error $e = \mu - \mu_d$, with $\mu_d = \alpha x_{1d} + x_{2d} + \cdots + x_{nd}$.

We design a control law u in terms of the properties of the solution of higher order ordinary differential equation. Consider a differential equation

$$a_r e^{(r)}(t) + a_{r-1}e^{(r-1)}(t) + \ldots + a_1\dot{e}(t) + a_0 e(t) = 0, \tag{13}$$

where r is the relative degree of the system. If a polynomial

$$p(s) = a_r s^r + a_{r-1}s^{r-1} + \ldots + a_1 s + a_0 \tag{14}$$

is Hurwitz, then the solution of differential equation (13) tends to zero if $t \to \infty$. In this case, for the purpose of designing the control law, an explicit relationship between input and output is required. To this end, we define a descent function as follows :

$$\begin{aligned}F(\mu, \mu_d, \dot{\mu}, \dot{\mu}_d, \ldots, \mu^{(n-1)}(t), \mu_d^{(n-1)}(t)) &= \left(\sum_{j=0}^{n-1} a_j(\mu - \mu_d)^{(j)}\right)^2 \\ &= \left(\sum_{j=0}^{n-1} a_j(e)^{(j)}\right)^2.\end{aligned} \tag{15}$$

By "Trajectory Following Method" [11], the control u is determined from the differential equation

$$\dot{u} = -\frac{\partial F}{\partial u} = -2a_{n-1}\left(\sum_{j=0}^{n-1} a_j (e)^{(j)}\right)\frac{\partial (e)^{(n-1)}}{\partial u}. \tag{16}$$

The control law in equation (16) is called the steepest descent control.

Calculate the time derivative of the descent function (15) along the trajectory of the extended system

$$\begin{aligned} \dot{x} &= \mathbf{A}x + \phi(y) + \theta\psi(y) + bu, && (17)\\ \dot{u} &= -2a_{n-1}\left(\sum_{j=0}^{n-1} a_j (e)^{(j)}\right)\frac{\partial (e)^{(n-1)}}{\partial u}. && (18) \end{aligned}$$

Then we have

$$\begin{aligned} \dot{F}(e,\dot{e},\ldots,e^{(n-1)}) &= 2\left(\sum_{j=0}^{n-1} a_j (e)^{(j)}\right)\left(\sum_{j=0}^{n-2} a_j (e)^{(j+1)}\right)\\ &+ 2a_{n-1}\left(\sum_{j=0}^{n-1} a_j (e)^{(j)}\right)\left(\frac{\partial a(e+\beta_d,\eta)}{\partial t} + \frac{\partial b(e+\beta_d,\eta)}{\partial t}u\right)\\ &- 2a_{n-1}\left(\sum_{j=0}^{n-1} a_j (e)^{(j)}\right) y_d^{(n)} - \left(\frac{\partial F}{\partial u}\right)^2. \end{aligned} \tag{19}$$

From equation (19), the value of the time derivative of the descent function (15) along the trajectory of (17) can not be guaranteed to be less than zero $t \geq 0$.

Now we modify the steepest descent control (16) by adding an artificial input v. Then the extended system (17) becomes

$$\begin{aligned} \dot{x} &= \mathbf{A}x + \phi(y) + \theta\psi(y) + bu,\\ \dot{u} &= -\frac{\partial F}{\partial u} + v. \end{aligned} \tag{20}$$

In the same way, let us calculate the time derivative of the descent function (15) along the trajectory of the extended system (20) yielding

$$\begin{aligned} \dot{F}(e,\dot{e},\ldots,e^{(n-1)}) &= 2\left(\sum_{j=0}^{n-1} a_j (e)^{(j)}\right)\left(\sum_{j=0}^{n-2} a_j (e)^{(j+1)}\right)\\ &+ 2a_{n-1}\left(\sum_{j=0}^{n-1} a_j (e)^{(j)}\right)\left(\frac{\partial a(e+\beta_d,\eta)}{\partial t} + \frac{\partial b(e+\beta_d,\eta)}{\partial t}u\right)\\ &- 2a_{n-1}\left(\sum_{j=0}^{n-1} a_j (e)^{(j)}\right) y_d^{(n)} - \left(\frac{\partial F}{\partial u}\right)^2 + \frac{\partial F}{\partial u}v. \end{aligned} \tag{21}$$

Consider equation (21). We will choose the artificial input v so that $\dot{F}(e, \dot{e}, \ldots, e^{(r)})$ be less then zero. We take

$$v = \frac{1}{\frac{\partial F}{\partial u}} \left(-k(e, \dot{e}, \ldots, e^{(n-1)}) \right), \tag{22}$$

where

$$\begin{aligned} k(e, \dot{e}, \ldots, e^{(n-1)}) &= 2 \left(\sum_{j=0}^{n-1} a_j (e)^{(j)} \right) \left(\sum_{j=0}^{n-2} a_j (e)^{(j+1)} \right) \\ &+ 2a_{n-1} \left(\sum_{j=0}^{n-1} a_j (e)^{(j)} \right) \left(\frac{\partial a(e+\beta_d, \eta)}{\partial t} + \frac{\partial b(e+\beta_d, \eta)}{\partial t} u \right) \\ &- 2a_{n-1} \left(\sum_{j=0}^{n-1} a_j (e)^{(j)} \right) y_d^{(n)}. \end{aligned} \tag{23}$$

Then

$$\dot{F}(e, \dot{e}, \ldots, e^{(n-1)}) = -\left(\frac{\partial F}{\partial u} \right)^2. \tag{24}$$

We have $\dot{F}(e, \dot{e}, \ldots, e^{(n-1)}) < 0$, if $\sum_{j=0}^{n-1} a_j (e_1)^{(j)} \neq 0$. Let $\sum_{j=0}^{n-1} a_j (e_1)^{(j)} = 0$. From equation (24) $\dot{F}(e, \dot{e}, \ldots, e^{(n-1)}) = 0$. Thus, the descent function (15) becomes minimum. The minimum value of descent function (15) is zero. Therefore $F(e, \dot{e}, \ldots, e^{(n-1)}) = 0$, then $\sum_{j=0}^{n-1} a_j (e)^{(j)} = 0$. Thus, we choose a_j, $j = 0, \ldots, n-1$ so that the polynomial $p(s) = a_0 + a_1 s + \cdots + a_{r-1} s^{n-2} + s^{n-1}$ is Hurwitz, then the error $e(t) \to 0$, if $t \to \infty$. Thus μ tends to μ_d if time $t \to \infty$. Hence the output of the original system $y = x_1$ tracks to the desired output $y_d(t)$.

Example 3.1

$$\begin{aligned} \dot{x_1} &= x_2 + x_1^2, \\ \dot{x_2} &= x_3 - u + x_1^2 + k_1 cos(t) \frac{x_1}{1+x_1^2}, \\ \dot{x_3} &= u - 2x_1^2 + k_2 sin(t)\ sin(x_1), \end{aligned} \tag{25}$$

$$y = x_1. \tag{26}$$

The zero dynamic system (25)-(26) is $\eta = \dot{\eta}$. Thus the system (25)-(26 is non-minimum phase. Now redefine the output : $z_1 = \alpha x_1 + x_2 + x_3$, with $0 < \alpha < 1$. Furthermore

$$\dot{z_1} = \alpha x_2 + (\alpha - 1)x_1^2 + x_3 + k_1 cos(t) \frac{x_1}{1+x_1^2} + k_2 sin(t)\ sin(x_1),$$

$$\ddot{z_1} = \alpha \dot{x_2} + 2(\alpha - 1)\dot{x_1} x_1 + \dot{x_3} + \frac{d}{dt}\left(k_1 cos(t) \frac{x_1}{1+x_1^2} \right) + \frac{d}{dt}\left(k_2 sin(t)\ sin(x_1) \right).$$

Thus the relative degree of the systems (25) with respect to the output z_1 is 2. If $z_1 = 0$, we have

$$\begin{aligned} \eta \dot{\eta} &\leq \frac{-\alpha}{|\alpha - 1|}\eta^2 + \frac{\eta^2}{|\alpha - 1|}\left(|k_1 cos(t)| + |k_2 sin(t)| \right) \\ &= \frac{\eta^2}{|\alpha - 1|}\left(-\alpha + (|k_1 cos(t)| + |k_2 sin(t)| \right). \end{aligned} \tag{27}$$

If $| \, k_1 cos(t) \, | + | \, k_2 sin(t) \, | \;\; < \;\; \alpha$, then $\eta\dot{\eta} \;\; < \;\; 0$. Thus system (25) with respect to the output z_1 is minimum phase. Let $y_d(t) = \pi/2$. Next, we choose z_{1d} so that if z_1 tracks $z_{1d}(t)$, then $y(t)$ tracks the desired output $y_d(t)$. By replacing x_1 with $x_{1d} = y_d = \pi/2$, we get $x_{2d} = -(\pi/2)^2$. By replacing x_2 with x_{2d}, we have the differential equation $\dot{x_3} - x_3 = -(\pi/2)^2 + k_2 sin(t) + k_1 \left(\frac{\pi/2}{1+(\pi/2)^2}\right) cos(t)$. Thus $x_{3d} = (\pi/2)^2 +$

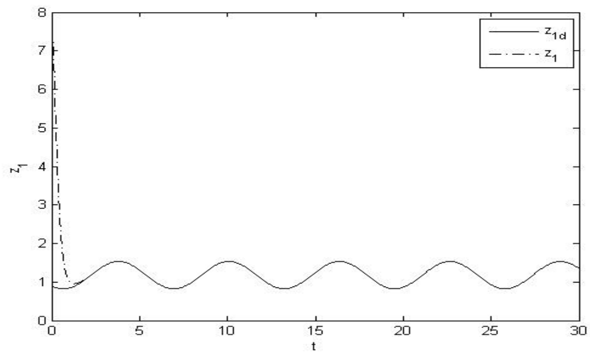

Figure 1: Output tracking z_1 *to* z_{1d}.

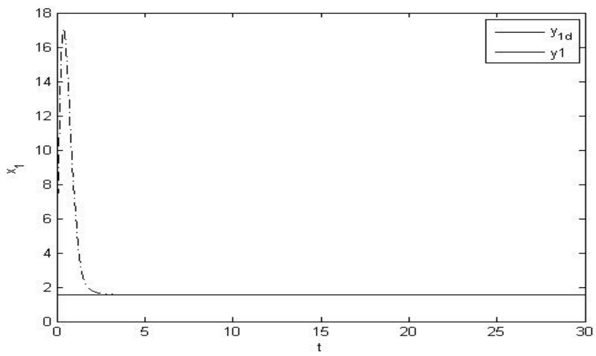

Figure 2: Output tracking y *to* $y_d = \pi/2$.

$0.5 sin(t) \left(k_1 \left(\frac{\pi/2}{1+(\pi/2)^2}\right) - k_2\right) + 0.5 cos(t) \left(-k_1 \left(\frac{\pi/2}{1+(\pi/2)^2}\right) - k_2\right)$. Now, $z_{1d} = \alpha x_{1d} + x_{2d} + x_{3d} = \alpha(\pi/2) + x_{3d}$. The modified steepest descent control with respect to the output z_1 is

$$\dot{u} = -\frac{\partial F}{\partial u} = -2a_2(a_0(z_1 - z_{1d}) + a_1(\dot{z_1} - \dot{z_{1d}}) + a_2(\ddot{z_1} - \ddot{z_{1d}}))(1-\alpha) + v, \qquad (28)$$

where v is the same as in equation (22). Simulation results are shown in Figure 1 and in Figure 2 for the constants $a_0 = 35$, $a_1 = 12$, $a_2 = 1$, $\alpha = 0.75$, $k_1 = 0.1$, $k_2 = 0.5$. The initial values $x_1(0) = 5$, $x_2(0) = 4$, $x_3(0) = 0$, $x_4(0) = 0$. In Figure 1, the output which has been selected so that the system becomes minimum phase tracks the desired output z_{1d}. In Figure 2, the output of the original system tracks the desired output $y_d = \pi/2$.

Example 3.2

$$\begin{aligned} \dot{x_1} &= x_2 - x_1^3, \\ \dot{x_2} &= x_3 - u + 2x_1^3, \\ \dot{x_3} &= \theta \sin(x_1) + u - 2x_1^3, \end{aligned} \tag{29}$$

$$y = x_1. \tag{30}$$

The zero dynamic system (25)-(26) is $\eta = \dot{\eta}$. Thus the system (25)-(26 is non-minimum phase. Now redefine the output : $z_1 = \alpha x_1 + x_2 + x_3$, with $0 < \alpha < 1$. The zero dynamic system (25)-(26) with respect to the output z_1 is

$$\dot{\eta} = \eta - \left(\frac{-\eta}{\alpha - 1}\right) - \left(\frac{-\eta}{\alpha - 1}\right)^3 + \theta \sin\left(\frac{-\eta}{\alpha - 1}\right).$$

We have

$$\begin{aligned} \eta\dot{\eta} &= \eta^2 + \frac{\eta^2}{\alpha - 1} + \frac{\eta^4}{(\alpha - 1)^3} + \eta\theta \sin\left(\frac{-\eta}{\alpha - 1}\right) \\ &\leq \eta^2 + \frac{\eta^2}{\alpha - 1} + \frac{\eta^4}{(\alpha - 1)^3} + |\eta||\theta| \left|\frac{-\eta}{\alpha - 1}\right| \\ &= \frac{\eta^2(|\theta| - \alpha)}{|\alpha - 1|} + \frac{\eta^4}{\alpha - 1}. \end{aligned} \tag{31}$$

If $|\theta| \leq \alpha$, then $\eta\dot{\eta} < 0$. Thus the system (29) with respect to the output z_1 is minimum phase. Let $y_d(t) = \pi/2$. By replacing x_1 with $x_{1d} = y_d = \pi/2$, we get $x_{2d} = (\pi/2)^3$. By replacing x_2 with x_{2d}, we have the differential equation $\dot{x_3} - x_3 = \theta$. Thus $x_{3d} = -\theta$. Now, $z_{1d} = \alpha x_{1d} + x_{2d} + x_{3d} = \alpha(\pi/2) + (\pi/2)^3 - \theta$. The modified steepest descent

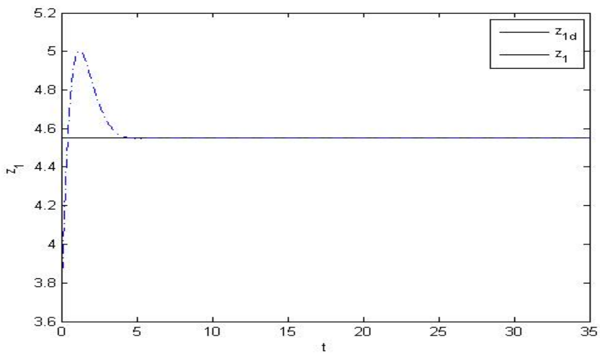

Figure 3: Output tracking z_1 *to* z_{1d}.

control with respect to the output z_1 is

$$\dot{u} = -\frac{\partial F}{\partial u} = -2a_2(a_0(z_1 - z_{1d}) + a_1(\dot{z_1} - \dot{z_{1d}}) + a_2(\ddot{z_1} - \ddot{z_{1d}}))(1 - \alpha) + v, \tag{32}$$

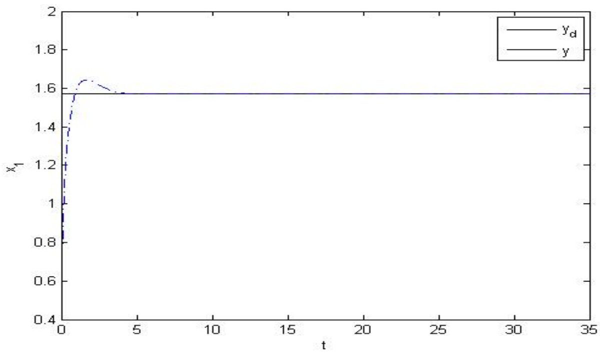

Figure 4: Output tracking y *to* $y_d = \pi/2$.

where v is the same as in equation (22). Simulation results are shown in Figure 3 and in Figure 4 for the constants $a_0 = 12$, $a_1 = 14$, $a_2 = 6$, $\alpha = 0.75$. The initial values $x_1(0) = 0,5$, $x_2(0) = 1$, $x_3(0) = 0$, $u(0) = 0$, $\theta(t) = 0.6$. In Figure 3, the output which has been selected so that the system become minimum phase tracks the desired output z_{1d}. In Figure 4, the output of the original system tracks the desired output $y_d = \pi/2$.

4 Conclusion

In this paper, we have designed the dynamic feedback control for output tracking of some class non-minimum phase nonlinear uncertain system (1)-(2). The design of the dynamic control is based on the modification of the steepest descent control. To apply the modified steepest descent control the system (1) is required to be minimum phase with respect to a new output, where the new output is the linear combination of the state variables. Furthermore, the new desired output will be set based on the desired output of the original system. By applying the modified steepest descent control, the system output tracks the desired output.

Acknowledgment

This research is supported by the Indonesian Directorate General of Higher Education (DIKTI).

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