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A NASC for Equicontinuous Maps for Integral Equations

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Abstract: We offer necessary and sufficient conditions for a mapping of the form

$$(P\phi)(t) = p(t) - \int_0^t C(t,s)g(s,\phi(s))ds$$

to send sets of bounded continuous functions on $[0,\infty)$ into equicontinuous sets. When that equicontinuity holds then one may study the problem of obtaining a bounded solution of the integral equation by means of a Schauder-type fixed point theorem. When the mapped sets are equicontinuous then we use Schaefer's fixed point theorem to show that we can obtain a bounded positive solution provided that we know that the resolvent kernel, $R(t,s)$, of C is non-negative and that

$$p(t) - \int_0^t R(t,s)p(s)ds$$

is bounded and positive, while $g(t,x)$ does not grow too fast near $x = 0$. The known literature shows that there are wide classes of important problems from applied mathematics and fractional equations for which these conditions hold. For those classes, the problem of obtaining a positive solution is largely solved when equicontinuity, characterized by our theorem, holds.

Keywords: *integral equations; compact maps; positive kernels; positive solutions; Schaefer's fixed point theorem.*

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