Capacity, Theorem of H. Brezis and F.E. Browder Type in Musielak–Orlicz–Sobolev Spaces and Application

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Abstract: The second section of this paper is devoted to the study of the capacity theory in Musielak–Orlicz–Sobolev space, we study basic's properties, including monotonicity, countable subadditivity and several convergence results, we prove that each Musielak-Orlicz-Sobolev function has a quasi-continuous representative. In the third section, we generalize the Theorem of H. Brezis and F.E. Browder in the setting of Musielak–Orlicz–Sobolev space $W^m L_\varphi(\Omega)$, which extends the previous result of H. Brezis and F.E. Browder [10]. In the fourth section, we make an application to an unilateral problem.

Keywords: *Musielak-Orlicz-Sobolev spaces; capacity; theorem of H. Brezis and F.E. Browder; unilateral problem.*

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1 Introduction

The theory of capacity and non-linear potential in the classical Lebesgue space $L^p(\Omega)$, was mainly studied by Maz'ya and Khavin in [17] and Meyers in [21]. These authors in their previous works have introduced the concept of capacity and non-linear potential in these spaces and provided very rich applications in functional analysis, harmonic analysis and in the theory of partial differential equations.

When we replace the spaces $L^p(\Omega)$ by the general one $L_A(\Omega)$ generated by an N-function, some fundamental properties are not satisfied, in particular, the reflexivity of

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