Nonlinear Dynamics and Systems Theory, **17** (1) (2017) 19–28

Integro-differential Equations, Compact Maps, Positive Kernels, and Schaefer's Fixed Point Theorem

T. A. Burton

Northwest Research Institute, 732 Caroline St., Port Angeles, WA 98362, USA

Received: December 18, 2016; Revised: January 23, 2017

Abstract: Integral equations offer a natural fixed point mapping, while an integro-differential equation

$$x'(t) = -g(t, x(t)) - \int_0^t A(t-s)f(x(s))ds$$

often prompts us to write it as an integral equation. This can be a mistake. We can convert it to an integral equation by a direct fixed point substitution which yields a very fixed point friendly equation. It is a natural sum of a continuous and compact map. Our first contribution here is to note that the direct fixed point process can change the continuous map to a compact map so that we have the sum of two compact maps and it is ready for Schauder's theorem instead of the much more complicated Krasnoselskii theorem which we usually expect to need. Schauder's theorem still requires that we find a self mapping set and that can be difficult. So we continue and combine the direct fixed point process with positive kernel theory so that we have an automatic *a priori* bound on all possible solutions of a homotopy equation. This gives us existence and boundedness of solutions of a wide class of problems from applied mathematics and it solves a classical problem which has been raised several times since 1960. See "Main note" following (3.1). That first contribution is a continuation of an earlier brief note in which we discovered that the direct fixed point process changed a Lipschitz map of an integro-differential equation into a contraction map, while here it changes a continuous map into a compact map.

Keywords: *integro-differential equations; compact maps; positive kernels; Schaefer's theorem; existence; uniqueness; fixed points.*

Mathematics Subject Classification (2010): 34A08, 34A12, 45D05, 45G05, 47H09.

* Corresponding author: mailto:taburton@olypen.com

