Searching Functional Exponents for Generalized Fourier Series and Construction of Oscillatory Functions Spaces

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Abstract: This paper is intended to provide a framework for further developments of the theory of generalized Fourier series of the form

$$\sum_{k=1}^{\infty} a_k \exp[i\lambda_k(t)], \ t \in \ R, \tag{1}$$

where $a_k \in \mathcal{C}$, $k \geq 1$, $\lambda_k : R \to R$, $k \geq 1$. Series of the form (1) will be called, in this paper, *series representing oscillatory functions*, by the last term understanding the sum of any series of the form (1), when convergent in some sense, classical or generalized, such as summability procedure or, in respect to a certain norm on the space of series, or in the associated function space of sums or generalized sums. A basic idea we follow is to start from linear spaces of series like (1), then to organize them by introducing a norm or a kind of convergence. The connection between a space of generalized trigonometric series of the form (1) and the space of functions resulting from introducing a topology/norm is our main objective. It is also emphasized that the preceding stages of Fourier analysis, i.e., the classical trigonometric series (the first stage) or the almost periodic functions (the second stage) are also parts of the *third stage* in the development of Fourier analysis. This study is based on classical theory of Fourier Analysis and on the theory of almost periodicity, as developed since 1920's to present. It is also based on methods and results of functional analysis.

Keywords: *generalized Fourier series; oscillatory functions; trigonometric series; almost periodic functions.*

Mathematics Subject Classification (2010): 42A32, 42A16, 42A75.

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