# Existence and Uniqueness Results by Progressive Contractions for Integro-Differential Equations

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**Abstract:** In this brief note we present a simple proof of global existence and uniqueness of a solution of an integro-differential equation

$$x'(t) = g(t, x(t)) + \int_0^t A(t-s)f(s, x(s))ds,$$

where $f$ and $g$ satisfy a Lipschitz condition with constant $K = K(t)$ where $K(t)$ is allowed to tend to infinity with $t$. The proof employs the idea of progressive contractions. It is a general fixed point theorem for differential equations.

**Keywords:** *fixed points; existence; uniqueness; progressive contractions; integro-differential equations.*

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## 1 Introduction

This is the third in a series of very short notes which we are constructing to illustrate the power, flexibility, and simplicity of a technique which we call *progressive contractions* to obtain a unique global solution of various kinds of differential and integral equations. We have applied the method to integral equations [4], fractional differential equations [6] of the type considered in [2], and integral equations of the Krasnoselskii type featuring a sum of two operators [5]. Each of the problems is of an essentially different type and the title of each note is chosen to allow interested readers to detect which subject is being treated.

In most of the existing literature investigators prove existence and uniqueness of solutions of differential equations by writing them as integral equations and applying

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