Extremal Mild Solutions for Nonlocal Semilinear Differential Equations with Finite Delay in an Ordered Banach Space

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Abstract: This paper is concerned with the existence and uniqueness of extremal mild solutions for nonlocal semilinear differential equations with finite delay in an ordered Banach space with the help of the monotone iterative technique based on lower and upper solutions. We use the theory of semigroup and measures of noncompactness to obtain the main results. The existence results are proved by assuming compact or non compact semigroup. An example is provided to illustrate the applicability of the main results.

Keywords: *initial value problem; finite delay; semigroup theory; monotone iterative technique; lower and upper solutions; Kuratowskii measure of noncompactness.*

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1 Introduction

In this paper, we consider the following nonlocal semilinear differential equations with finite delay in an ordered Banach space:

$$\begin{cases} \frac{d}{dt}x(t) = & Ax(t) + f(t, x_t, Bx(t)), \quad t \in J = [0, b], \\ x(t) = & \phi(t) + g(x)(t), \quad t \in [-a, 0], \end{cases} \tag{1}$$

where the state $x(\cdot)$ takes values in the Banach space X endowed with norm $\| \cdot \|$; $A: D(A) \subset X \to X$ is a closed linear densely defined operator and an infinitesimal generator of strongly continuous semigroup $\{T(t)\}_{t \geq 0}$ of bounded linear operator in X;

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