Nonlinear Dynamics and Systems Theory, 16 (2) (2016) 192-205



## Boundedness and Square Integrability of Solutions of Nonlinear Fourth Order Differential Equations

M. Remili<sup>\*</sup> and M. Rahmane

Department of Mathematics, University of Oran 1 Ahmed Ben Bella, Algeria.

Received: March 13, 2015; Revised: March 28, 2016

**Abstract:** Sufficient conditions for the boundedness of the solutions to a certain nonlinear fourth order differential equation are given by means of the Lyapunov's second method. We also give criteria for square integrability of solutions and their derivatives. Example is given to illustrate our results.

**Keywords:** boundedness; stability; Lyapunov function; fourth-order differential equations;  $L^2$  solutions; square integrable.

Mathematics Subject Classification (2010): 34D20, 34C11.

## 1 Introduction

Higher-order nonlinear differential equations are frequently encountered in mathematical models of most dynamic processes in electromechanical systems in physics and engineering. The notions of stability and boundedness of solutions are fundamental in the theory and application of differential equations. In this way, both concepts lead to the real world applications. Many results relative to stability, boundedness, square integrability of solutions to differentiel equations have been obtained. See for instance ([1]-[42]). In discussing stability and boundedness of a nonlinear differential system, Lyapunov's direct method perhaps is the most effective method. Numerous methods have been proposed in the literature to derive suitable Lyapunov functions, but finding a proper Lyapunov's function in general is a big challenge.

The study of fourth order nonlinear differential equations has attracted the interest of many researchers. Many results concerning the stability and boundedness of solutions of fourth order differential equations have been obtained in view of various methods, especially, Lyapunov's method, see, the book of Reissig et al. [28] as a survey and the

<sup>\*</sup> Corresponding author: mailto:remilimous@gmail.com

<sup>© 2016</sup> InforMath Publishing Group/1562-8353 (print)/1813-7385 (online)/http://e-ndst.kiev.ua192