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Entropy Solutions of a Quasilinear Degenerated Elliptic Unilateral Problems With L^1 Data and Without Sign Condition

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Abstract: In this paper, we will be concerned with the existence of solutions for strongly nonlinear degenerated elliptic unilateral problems associated with the equation $A(u)+g(x,u,\nabla u)+H(x,\nabla u)=f$, where A is Leray-Lions operator acting from $W_0^{1,p}(\Omega,w)$ to its dual. On the nonlinear term $g(x,s,\xi)$, we assume growth condition on ξ and without assuming the sign condition on s, while the function $H(x,\xi)$, which induces a convection term, is only growing at most as $|\xi|^{p-1}$. The right-hand side f belongs to $L^1(\Omega)$.

Keywords: *weighted Sobolev spaces; quasilinear degenerated unilateral problems; non-variational inequalities.*

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1 Introduction

Let Ω be a bounded open subset of $\mathbb{R}^N$ $(N \geq 2)$, $1 < p < \infty$ and $w = \{w_i(x), i = 0,\dots,N\}$ be a vector of weight functions on Ω, i.e. each $w_i(x)$ is a measurable strictly positive function on Ω, satisfying some integrability conditions. Let $X = W_0^{1,p}(\Omega,w)$ be the weighted Sobolev space associated with the vector w. Consider the following non-linear Dirichlet problem

$$\begin{cases} A(u)+g(x,u,\nabla u)+H(x,\nabla u)=f \quad \text{in } \mathfrak{D}'(\Omega), \\ u \in W_0^{1,p}(\Omega,w),\ g(x,u,\nabla u)\in L^1(\Omega),\ H(x,\nabla u)\in L^1(\Omega), \end{cases} \tag{1}$$

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