



Generalized Iterative Methods for Caputo Fractional Differential Equations via Coupled Lower and Upper Solutions with Superlinear Convergence

M. Sowmya¹ and A.S. Vatsala^{2*}

¹ *R. V. College of Engineering, Bengaluru, Karnataka 560059, India*

² *Department of Mathematics, University of Louisiana at Lafayette, Lafayette, Louisiana 70504, USA*

Received: November 29, 2014; Revised: April 2, 2015

Abstract: Existence of coupled lower and upper solutions for nonlinear differential equations guarantees the existence as well as interval of existence of the solution. In this work, a methodology has been developed to compute coupled lower and upper solutions using natural lower and upper solutions by iterative methods. Further, using the computed lower and upper solutions, sequences are developed which converge uniformly and monotonically to the unique solution. In addition, it has been shown that the convergence of these sequences is superlinear. Further the convergence of the sequences is accelerated by Gauss-Seidel method. Finally, some numerical examples are presented.

Keywords: *Caputo fractional differential equation; superlinear convergence.*

Mathematics Subject Classification (2010): 34A08, 34A12.

1 Introduction

It is well-known that qualitative and quantitative properties of fractional differential equations are very useful in applications. In addition, fractional differential equations in several situations have proved to be better and more economical models than their counterpart with integer derivatives. For details see [5, 9, 11] and the references therein. In the past thirty years there has been a rapid development in the qualitative study of fractional differential equation such as existence, uniqueness and stability results due to its applications. In particular, it has been very useful in biological sciences such

* Corresponding author: <mailto:vatsala@louisiana.edu>