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A Fractional Order $PI^{\alpha}D^{\beta}$ Control of the Nonlinear Systems

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Abstract: This paper studied the implementation of fractional order $PI^{\alpha}D^{\beta}$ controller for the control of an induction motor (IM). The perfection of the system performance in terms of response time and robustness is illustrated by adjusting the fractional order integral action and derivative action. A comparative study with a conventional PID controller is carried out. The observer is simple and robust, and suitable for online implementation for induction motor. Simulation tests under load disturbances and parameter uncertainties are provided to evaluate the consistency and performance of the proposed control technique.

Keywords: *conventional controller; fractional order controller; induction motor IM; electromagnetic torque and flux control.*

Mathematics Subject Classification (2010): 26A33, 93C10.

1 Introduction

The conventional PID controller is widely used in automatic and especially in industry because of its simplicity but due to the complexity of the controlled systems and parametric variations, the PID controller can not reach the desired performance control where the use of fractional order controller with integral action and derivative action, non-integer order.

The fractional order $PI^{\alpha}D^{\beta}$ controller is an improved version of the conventional PID controller. It allows two degrees of freedom to better adjust the dynamic properties

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of the system and can control non-integer order systems [1–4]. The fractional $PI^\alpha D^\beta$ controller is less sensitive to parameter variations of the system, it is a robust controller.

The fractional control was developed by mathematicians in the eighties [5,6]. In the last decade, the calculation of fractional order is applied to each field of engineering. It made a profound impact in the theories of control [7–12].

There are several methods of approximation of the derivative and integral fractional controller [13–15]. The methods of approximations are distinguished by the entire model obtained being continuous or discrete. Researches are ongoing to improve and adjust the controller parameters to expand the scope of application of the fractional control.

In this paper, we will determine the theory of fractional $PI^\alpha D^\beta$ controller for controlling an induction machine. A parametric variation of the controller is used to determine the influence of fractional controller of control system with and without the presence of disturbance on the system [16–18].

The paper is organized as follows: In Section 2 the Induction Machine modeling is presented. In Section 3, synthesis of the IM controllers is studied. In Section 4, implementation of fractional order controller is considered. In Section 5 the simulation results are presented and discussed, and finally in Section 6 conclusions are drawn.

2 IM Modelling

Prior to the IM equating, some assumptions are considered [19,20]:

- The gap is constant.
- The Hysteresis, the saturation and the eddy currents are neglected.
- The magneto-motive forces generated by the stator and rotor phases have a sinusoidal distribution.

(a) Mathematical model for the IM.

- *Electrical equations:*

$$\begin{aligned} V_{dS} &= R_S I_{dS} + \tfrac{d\phi_{dS}}{dt} - \omega_S \phi_{qS}, \quad V_{qS} = R_S I_{qS} + \tfrac{d\phi_{qS}}{dt} + \omega_S \phi_{dS}, \\ 0 &= R_r I_{dr} + \tfrac{d\phi_{dr}}{dt} + \omega_{Sl} \phi_{qr}, \quad 0 = R_r I_{qr} + \tfrac{d\phi_{qr}}{dt} + \omega_{Sl} \phi_{qr}, \end{aligned} \tag{1}$$

where

$$\begin{aligned} \phi_{dS} &= L_S I_{dS} + L_m I_{dr}, \quad \phi_{qS} = L_S I_{qS} + L_m I_{qr}, \\ \phi_{dr} &= L_m I_{dS} + L_r I_{dr}, \quad \phi_{qr} = L_m I_{qS} + L_r I_{qr}, \end{aligned} \tag{2}$$

$$\omega_S = 2\pi f = \frac{d\theta_S}{dt}, \tag{3}$$

$$\omega_{Sl} = \omega_S - \omega_r, \tag{4}$$

with:
L_S: Stator proper cyclical inductance,
L_r: Rotor proper cyclical inductance,
L_m: Cyclical mutual inductance between stator and rotor,
ω_S: Synchronization speed,
ω_{Sl}: Sliding angular velocity.

- *Mechanical equation:*

The mechanical equation is defined by:

$$C_{em} = \frac{3}{2}p\frac{M_{Sr}}{L_r}\left(\phi_{dr}I_{qS} - \phi_{qr}I_{dS}\right). \tag{5}$$

- *Torque equation:*

The orientation of the (dq) frame with the d axis associated with the rotor flux allows writing: $\phi_{dr} = \phi_r$ and $\phi_{qr} = 0$. Thanks to this flux orientation, which allows a high starting torque, the torque expression can be simplified as follows:

$$C_{em} = \frac{3}{2}p\frac{M_{Sr}}{L_r}\phi_{dr}I_{qS}. \tag{6}$$

3 Synthesis of the IM Controllers

The IM state equations are as follows:

$$\frac{dI_{Sd}}{dt} = -\frac{1}{\sigma L_S}\left(R_S + \frac{M_{Sr}^2 R_r}{L_r^2}\right)I_{Sd}+$$

$$\omega_S I_{Sq} + \frac{1}{\sigma L_S}\frac{M_{Sr}R_r}{L_r^2}\phi_{rd} + \frac{1}{\sigma L_S}\frac{M_{Sr}}{L_r}p\Omega_m\phi_{rq} + \frac{1}{\sigma L_S}V_{Sd}, \tag{7}$$

$$\frac{dI_{Sq}}{dt} = \frac{1}{\sigma L_S}\left(R_S + \frac{M_{Sr}^2 R_r}{L_r^2}\right)I_{Sq}+$$

$$\omega_S I_{Sd} + \frac{1}{\sigma L_S}\frac{M_{Sr}R_r}{L_r^2}\phi_{rq} - \frac{1}{\sigma L_S}\frac{M_{Sr}}{L_r}p\Omega_m\phi_{rd} + \frac{1}{\sigma L_S}V_{Sq}, \tag{8}$$

$$\frac{d\phi_{rd}}{dt} = \frac{M_{Sr}R_r}{L_r}I_{Sd} - \frac{R_r}{L_r}\phi_{rd} + \left(\omega_S - p\Omega_m\right)\phi_{rq}, \tag{9}$$

$$\frac{d\phi_{rq}}{dt} = \frac{M_{Sr}R_r}{L_r}I_{Sq} - \frac{R_r}{L_r}\phi_{rq} - \left(\omega_S - p\Omega_m\right)\phi_{rd}, \tag{10}$$

$$\frac{\Omega_m}{dt} = \frac{3}{2}\frac{M_{Sr}P}{L_r J}\left(\phi_{rd}I_{Sq} - \phi_{rq}I_{Sd}\right) - \frac{F}{J}\Omega_m - \frac{1}{J}C_r, \tag{11}$$

while: $\sigma = 1 - \frac{M_{Sr}^2 R_r}{L_S L_r}$.

(a) Control loop of the rotor flux.

The decoupling allowed by the oriented flux and the relation (3) can give

$$\frac{d\phi_{rd}}{dt} = \frac{M_{Sr}R_r}{L_r}I_{Sd} - \frac{R_r}{L_r}\phi_{rd}. \tag{12}$$

Wherein the direct stator current expression is:

$$I_{Sd} = \frac{1}{M_{Sr}}\left(\phi_{rd} + \frac{L_r}{R_r}\frac{d\phi_{rd}}{dt}\right). \tag{13}$$

Let $T_r = \frac{L_r}{R_r}$ be the rotor time constant and $T_S = \frac{L_S}{R_S}$ be the stator time one. The relations (7) and (13) can lead to:

$$V_{Sd} = \frac{R_S}{M_{Sr}}\left(\phi_{rd} + (T_S + T_r)\frac{d\phi_{rd}}{dt}\right) + \sigma T_S T_r \frac{d^2\phi_{rd}}{dt^2} - \omega_S \sigma L_S I_{Sq} = V_{Sdf} + V_{Sdc}. \quad (14)$$

To ensure the decoupling between the two axes, the term V_{Sdc} must be compensated:

$$V_{Sdf} = \frac{R_S}{M_{Sr}}\left(\phi_{rd} + (T_S + T_r)\frac{d\phi_{rd}}{dt}\right) + \sigma T_S T_r \frac{d^2\phi_{rd}}{dt^2}, \quad V_{Sdc} = -\omega_S \sigma L_S I_{Sq}. \quad (15)$$

The system transfer function is:

$$G_{flux}(p) = \frac{\phi_{rd}(p)}{V_{Sdf}(p)} = \frac{M_{Sr}}{R_S}\frac{1}{1 + (T_S + T_r)p + \sigma T_S T_r p^2}. \quad (16)$$

Let p_1 and p_2 be the denominator roots such that $p_2 \succ\succ p_1$, where $p_1 = \frac{\sigma T_S T_{Sq}}{T_S + T_{Sq} + \Delta}$, $p_2 = \frac{\sigma T_S T_{Sq}}{T_S + T_{Sq} - \Delta}$.

The flux error is $\epsilon = e_{2\_PI} = \phi_{rd\_rf} - \phi_{rd}$. The following figure shows the block diagram of the flux control loop.

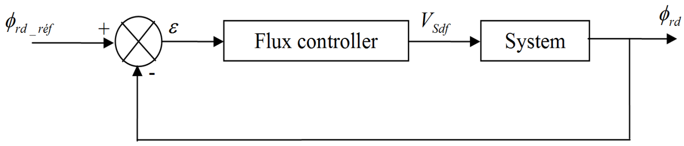

Figure 1: Flux control loop.

(b) Control loop of the electromagnetic torque.

Considering that the flux response is faster than the torque one, the flux reaches its final value $\phi_{rd} = \phi_{rd0}$, and the expression of the torque could be given by the following:

$$C_{em} = \frac{3}{2}\frac{M_{Sr}P}{L_r}\phi_{rd0} I_{Sd}. \quad (17)$$

The voltage equation V_{Sq} becomes:

$$V_{Sd} = R_S I_{Sq} + \sigma L_S \frac{dI_{Sq}}{dt} + \phi_{rd}\omega_S \frac{M_{Sr}}{L_r} + \sigma L_S \omega_S I_{Sd}. \quad (18)$$

Let

$$V_{Sq} = V_{Sqt} + V_{Sqc}. \quad (19)$$

The V_{Sqc} component represents a decoupling term that we have to compensate,

$$V_{Sqc} = \phi_{rd}\omega_S \frac{M_{Sr}}{L_r} + \sigma L_S \omega_S I_{Sd}, \quad (20)$$

$$V_{Sqt} = R_S I_{Sq} + \sigma L_S \frac{dI_{Sq}}{dt}. \quad (21)$$

The system transfer function becomes:

$$G_{cem}(p) = \frac{C_{em}(p)}{V_{Sqt}(p)} = \frac{3M_{Sr}P\phi_{rd0}}{2L_r R_S(1 + \sigma T_S p)}. \quad (22)$$

The flux error is $\epsilon = e_{1\_PI} = C_{em\_rf} - C_{em}$. The following figure shows the block diagram of the torque control loop.

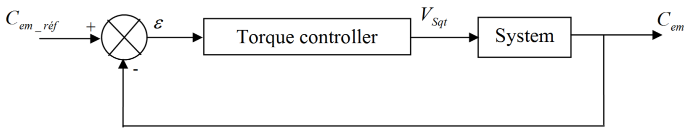

Figure 2: Torque control loop.

4 Implementation of Corrective Fractional Order

The simulation part is usually performed by integer order of finite dimension. So it is necessary to replace the transfer functions of non-integer order by the transfer functions of integer order. The methods of approximations are distinguished by the entire model obtained, being continuous or discrete.

(a) Continuous Approximation Methods: singularity function.

There are several approximation methods analog continuous (or frequency) for the fractional operators existing in the literature [21, 22]. These methods are based on the continuous model, such as the approximation of fractional order model by a continuous rational model.

The method consists in replacing the derivative operator S^n by a transmittance, where poles and zeros are related by a recurrence relation. To replace S^n by an entire model, it is necessary to apply the following approximations:

- Approximation in a frequency band $[\omega_B; \omega_H]$ of non-integer operator by a non-integral model $S^n_{[\omega_B;\omega_H]}$.
- Approximation of the non-integer model obtained by an entire model.

The approximation methods are: SFEC approximation Method (Fractional Expansion Continues), Oustaloup approximation method [23], Charef approximation method [24], other methods (Carlson, Matsuda, Roy Wang, ...). In the following we will define the Charef method as an example.

- *Approximation of fractional order integration.*

The transfer function of the fractional order integrator is given by the following irrational function [4, 25]:

$$H_1(p) = \frac{1}{P^\alpha}, \tag{23}$$

where α is a positive number $0 \prec \alpha \prec 1$ and $p = j\omega$ is the complex frequency. This operator may be approximated in a given frequency band $[\omega_B; \omega_H]$ by:

$$H_1(p) = \frac{k_1}{\left(1 + \frac{P}{\omega_C}\right)^\alpha} = k_1 \frac{\prod_{i=0}^{N-1}(1 + \frac{P}{\tau_i})}{\prod_{i=0}^{N}(1 + \frac{P}{P_i})}. \tag{24}$$

For systems with integrator: The transfer function of the fractional order integrator is given by the following irrational function [26]:

$$H_1(p) = \frac{1}{P^\alpha} = \frac{1}{P} P^{1-\alpha}. \tag{25}$$

Thus

$$H_1(p) = \frac{k_D}{P}\frac{\prod_{i=0}^{N}(1+\frac{P}{T_i})}{\prod_{i=0}^{N}(1+\frac{P}{Z_i})}, \tag{26}$$

- *Approximation of fractional order differentiation*

The transfer function of fractional order differentiator is given by the following irrational function:

$$H_D(p) = P^{\beta}, \tag{27}$$

where β is a positive number $0 \prec \beta \prec 1$ and $p = j\omega$ is the complex frequency. This operator may be approximated in a given frequency band $[\omega_B; \omega_H]$ by:

$$H_D(p) = \frac{k_D}{\left(1+\frac{P}{\omega_C}\right)^{\beta}} = k_D\frac{\prod_{i=0}^{N}(1+\frac{P}{T_i})}{\prod_{i=0}^{N}(1+\frac{P}{Z_i})}. \tag{28}$$

(b) Adjusting the parameters of the controller PI^{α}.

- *Adjustment of parameters k_p and k_i*

For flow control, we will apply the compensation method for compensating the slow term and make the system faster, hence the use of a corrector PI. This type of corrector is generally used for the first order systems such as the torque control. The adjustment of parameters k_p and of fractional order PI^{α} control is done with $\alpha = 1$, which means adjusting the parameters of a simple classical PI controller. To compensate for the dominant pole, we will use a fractional order PI^{α} controller. The shape of the fractional order PI^{α} controller, including a fractional integrator of order α, such as $0 \prec \alpha \prec 1$, see [27]. The transfer function of fractional order control is given by:

$$C(p) = k_p\left(1 + k_i\frac{1}{P^{\alpha}}\right). \tag{29}$$

- ***Flow Control:***

The transfer function of open loop flow control is:

$$H_0(p) = G_f(p)C(p) = \frac{\phi_{rd}(p)}{\epsilon(p)} = \frac{M_{Sr}}{R_S}.k_i.\frac{1}{(1+p_1p)(1+p_2p)}.\frac{1+\frac{k_p}{k_i}p}{p}. \tag{30}$$

Using the compensation method of dominant pole (offset slow time constant) is to make the system faster. The transfer function in simplified open loop is given by:

$$H_0(p) = \frac{M_{Sr}}{R_S}.k_i.\frac{1}{p(1+p_1p)}. \tag{31}$$

The transfer function of the closed loop is:

$$H_F(p) = \frac{1}{1+\frac{R_S}{M_{Sr}k_i}p+\frac{R_S}{M_{Sr}k_i}p_1p^2} = \frac{1}{1+\frac{2z}{\omega_n}p+\frac{1}{\omega_n^2}p^2} \tag{32}$$

with $k_p = k_ip_2$, $k_i = \frac{R_S}{M_{Sr}}.\frac{\omega_n}{2z}$ and $\omega_n = \frac{1}{2zp_1}$.

Choice of parameters z and ω_n.

- A good starting point is to clean the pulse ω_n equal to the open-loop process.
- The excess is determined by the value $z = 0.7$ providing a good response time.
- To have a positive adjustment we need $k_i \succ 0$.

- ***Electromagnetic torque control:***

The transfer function in open lopp is:

$$H_0(p) = G_{cem}(p)C(p) = \frac{\phi_{rd}(p)}{\epsilon(p)} = \frac{3M_{Sr}P\phi_{rd0}}{2L_rR_S}.k_i.\frac{1}{(1+\sigma T_S p)}.\frac{1+\frac{k_p}{k_i}p}{p}. \tag{33}$$

The transfer function in simplified open loop is given by:

$$H_0(p) = \frac{3M_{Sr}P\phi_{rd0}}{2L_rR_S}.k_i.\frac{1}{p}. \tag{34}$$

The transfer function of the closed loop is:

$$H_F(p) = \frac{1}{1+\frac{1}{k.k_i}p}. \tag{35}$$

Choice of parameters: k_i ***and*** τ.

- A good starting point is to take the constant τ equal to the process time.
- To have a positive adjustment we need $k_i \succ 0$.

- *Adjustment parameter* α

To adjust the parameters α or (β) by minimizing a performance criterion is the integral square error (ISE). The integral square error (ISE) is given by:

$$J = \int_0^\infty [e(t)]^2\, dt = \frac{1}{2\pi j}\int_{-j\infty}^{+j\infty} E(p)E(-p)dp. \tag{36}$$

The error signal $E(p)$ is obtained as:

$$E(p) = \frac{R(p)}{1+C(p)G(p)}, \tag{37}$$

where $R(p)$ is a unit step input

$$R(p) = \frac{1}{p}. \tag{38}$$

- *Hall-Sartorius method*

To calculate ISE we use the Hall-Sartorius method. It is to minimize the integral squared error of a loop with an entry level system

$$J = \frac{1}{2\pi j}\int_{-j\infty}^{+j\infty} \frac{N_E(p)N_E(-p)}{D_E(p)D_E(-p)}dp, \tag{39}$$

$$N_E(p) = b_0 + b_1 p + b_2 p^2 + \ldots + b_{n-1} p^{n-1}, \tag{40}$$

$$D_E(p) = a_0 + a_1 p + a_2 p^2 + \ldots + a_{n-1} p^{n-1} + a_n p^n, \tag{41}$$

$$N_E(p) = c_0 + c_1 p^2 + c_2 p^4 + \ldots + c_{n-1} p^{2(n-1)}, \tag{42}$$

or the general formula

$$J = \frac{(-1)^{n-1}}{2} \cdot \frac{det(\Delta_n^N)}{det(\Delta_n^D)} \tag{43}$$

with $\Delta_n^D \in \Re^{(n+1)(n+1)}$,

$$Delta_n^D = \frac{(-1)^{n-1}}{2} \cdot \frac{det(\Delta_n^N)}{det(\Delta_n^D)}, \tag{44}$$

$$\Delta_n^D = \begin{bmatrix} a_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_2 & a_1 & a_1 & 0 & 0 & 0 & 0 & 0 \\ . & . & . & . & . & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & a_n & a_{n-1} & a_{n-2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_n \end{bmatrix} \tag{45}$$

and $\Delta_n^N \in \Re^{(n)(n)}$. The matrix Δ_n^N is obtained by removing the last column and last row of the matrix Δ_n^N and replacing the last column of this matrix by the following vector:

$$\Delta_n^D = \begin{bmatrix} c_0 & c_1 & a_2 & \ldots & a_1 \end{bmatrix}. \tag{46}$$

The smallest index J of the criterion ISE, $J = 0.5094$ is calculated with $\alpha = 0.92$ for the flow control, and $J = 0.0054$ is obtained with $\alpha = 0.65$ for the electromagnetic torque control. The integrator and the differentiator to the fractional order controller C(p) are approximated in the frequency band $[\omega_B; \omega_H] = [0.1\omega_B; 10.\omega_H]$ with a frequency $\omega_{max} = 100\omega_h$ and an approximation error $y = 1dB$.

Hence, the controller fractional order $PI^{0.65}$ is given by:

$$C(p) = 286.308 \left(1 + \frac{1.4054}{p} \cdot \frac{\prod_{i=0}^{3} (1 + \frac{p}{0.2215.(433.873)^i})}{\prod_{i=0}^{3} (1 + \frac{p}{1.8556.(352.1189)^i})} \right). \tag{47}$$

The controller fractional order $PI^{0.65}$ is given by:

$$C(p) = 0.0351 \left(1 + \frac{64.9076}{p} \cdot \frac{\prod_{i=0}^{6} (1 + \frac{p}{2.2624.10^{-4}.(28.84)^i})}{\prod_{i=0}^{6} (1 + \frac{p}{2.9058.10^{-4}(22.84)^i})} \right). \tag{48}$$

Table 1 summarizes some performance characteristics of the conventional control system and fractional order in terms of the cutoff frequency $\omega_u(rad/s)$, response time $t_r(s)$, Gain Margin $GM(dB$, Phase Margin $PM(deg)$, and overshoot $D\%$.

5 Simulation Results

The following figures are determined using the Matlab / Simulink software to demonstrate the performance of the fractional order control. The performance of the control technique is defined by:

	cont	ω_u	t_r	GM	PM	$D\%5$
Flux control	PI	120	0.1805	-	65.5	4.54
	$PI^{0.65}$	120	0.1805	-	65.5	4.54
Torque control	PI	1	3.29	-	90	-
	$PI^{0.92}$	0.984	2.8	-	96.4	-

Table 1: Characteristics of performance for $(PI;\, PI^{\alpha})$.

- Stability in steady state.
- Response quickness.
- A relatively small static error.

The simulation is performed with unloading start, at t=60s rotation is reversed, then a load torque $C_r = 20Nm$ is introduced at t=100s.

Figure 3 represents the evolution of the electromagnetic torque considered, real and reference of the asynchronous motor in the presence of radial force $C_r = 20Nm$ t=100s. It is noted that the electromagnetic torque does not admit oscillations and reaches steady operation with a response time $tr_{PI} = 3.92s$ et $tr_{PI^{0.92}} = 2.8s$. The machine answers successfully to the inversion of its direction of rotation.

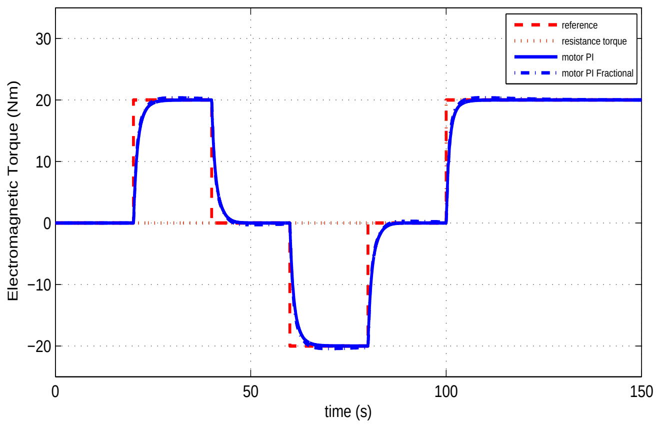

Figure 3: Evolution of the electromagnetic torque(- - PI; - $PI^{0.92}$).

Figure 4 shows the influence of controls applied on the response of flow along the two axes (d, q):

- Along the axis (d): the fractional order control is less sensitive to the reversal of direction of rotation or the introduction of load than the PI controller.
- Along the axis (q): the flow is zero regardless of the order.

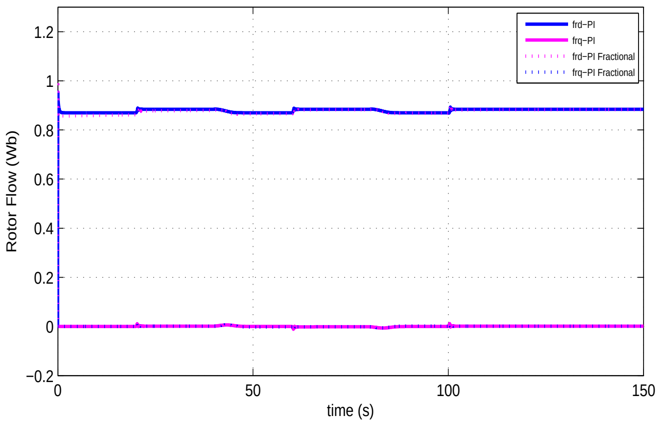

Figure 4: Rotor flux response.

Changes in the motor flux demonstrate the robustness of the control slide, it follows exactly the desired set point, with overshoot negligible, see Table 1, and without static error even for the impact load torque or reversal of direction of rotation. The evolution of direct rotor flux is not a static error with short response time.

Figure 5 is a representation of the evolution of the speed of asynchronous techniques for both commands. The response speed of the MAS shown in Figure 5 is similar to that of a first order system without overshoot, steady and stable with a response time of the order of 5.36s for the speed defined by the $PI^{0.65}$ controller and 5.63s for the speed determined by the classical PI controller. The evolution of the velocity shows at t = 100s the robustness of the fractional order control to the introduction of charging. $S_{PI^{0.65}} = (4.8\%)S_{PI}$.

To demonstrate the performance of control system by fractional order control, we will vary the time constant and process gain for the torque control in closed loop. And, we will vary the damping factor for the flux control in closed loop.

Figures 6 and 7 represent the influence of the variation of time constant. It is assumed that the gain is fixed at its nominal value K_{nom}. To study the influence of the variation of the time constant τ the parameter τ is varied around its nominal value. The results show that:

- the response time Defines by the fractional order $PI^{0.92}$ controller is still less than the response time defines by the conventional controller for different values of the time constant τ.
- the overshoot is insensitive to the variation of the time constant τ.
- the servo by the $PI^{0.92}$ controller, ensure the desired specifications with the presence of a very important property of robustness.

Figures 8 and 9 represent the influence of the variation of process gain. It is assumed that the time constant is fixed at its nominal value τ_{nom}. To study the influence of the variation of the process gain K the parameter K is varied around its nominal value. The results show that:

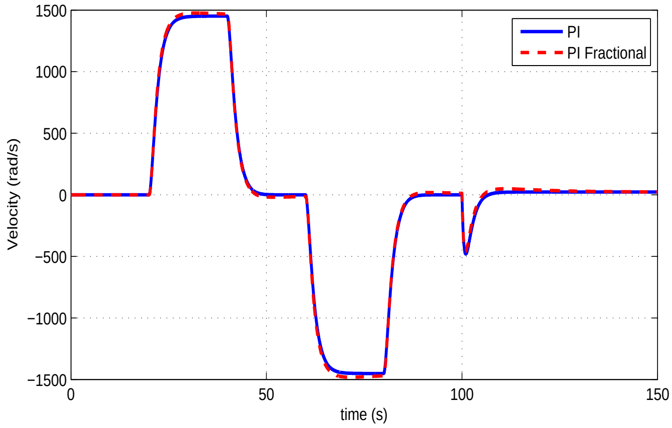

Figure 5: Evolution of the speed (- - PI; - $PI^{0.65}$).

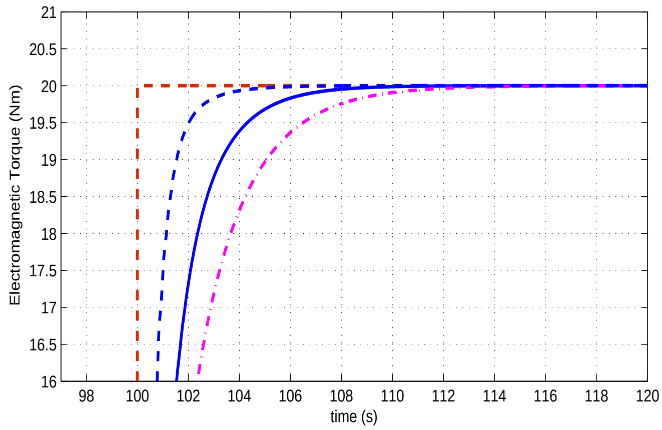

Figure 6: Evolution of the electromagnetic torque for different values of time constant τ, (- $\tau = \tau_{nom}$; -.- $\tau = 150\%\tau_{nom}$; - - $\tau = 50\%\tau_{nom}$) (conventional PI).

- the response time defined by the fractional order $PI^{0.92}$ controller is still less than the response time defined by the conventional controller for different values of the process gain K.
- the overshoot is insensitive to the variation of the process gain K.
- the servo by the $PI^{0.92}$ controller, ensures the desired specifications with the presence of a very important property of robustness.

Figures 10 and 11 show the impact of the variation of the damping factor (m) on the flux response along the axe (d). It was found that, the rise in response to the desired value, the higher the damping factor (m).

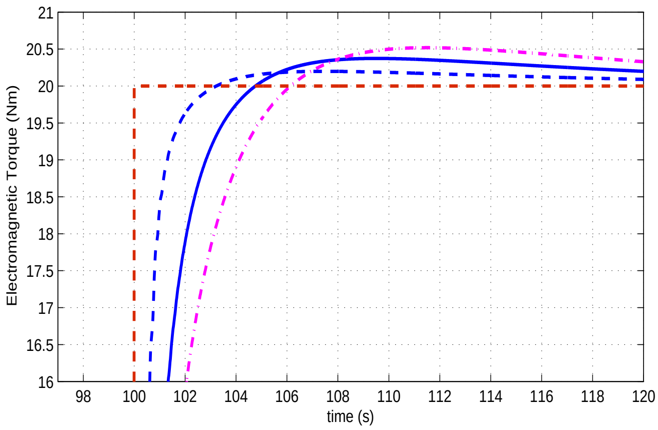

Figure 7: Evolution of the electromagnetic torque for different values of time constant τ, (- $\tau = \tau_{nom}$; -.- $\tau = 150\%\tau_{nom}$; - - $\tau = 50\%\tau_{nom}$) $(PI^{0.92})$.

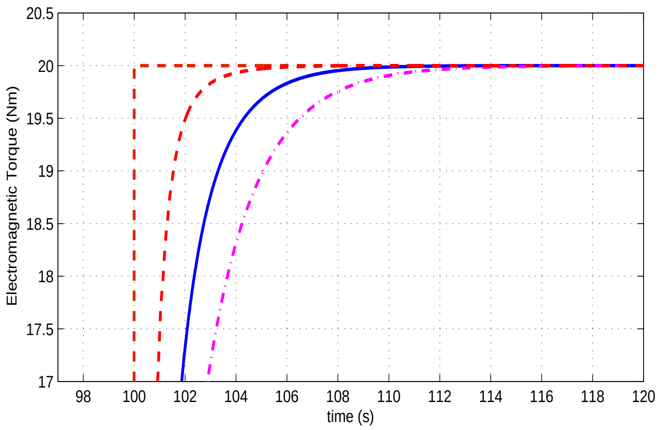

Figure 8: Evolution of the electromagnetic torque for different values of process gain K, (- $K = K_{nom}$; -.- $K = 150\%K_{nom}$; - - $K = 50\%K_{nom}$) (conventional PI).

- the response time defined by the fractional order controller is still less than the response time defined by the conventional controller for different values of the damping factor m. $tr^{PI}(m = 0.5) = 0.071s$; $tr^{PI}(m = 0.7) = 0.18s$ and $tr^{PI}(m = 1.2) = 1.797s$. $tr^{PI^{0.65}}(m = 0.5) = 0.069s$; $tr^{PI^{0.65}}(m = 0.7) = 0.156s$ and $tr^{PI^{0.65}}(m = 1.2) = 1.687s$
- the overshoot of flux defined by the fractional order $PI^{0.65}$ controller is less sensitive than the overshoot defined by the conventional controller for different values of the damping factor m. For example, m=0.5: $D^{PI}(m = 0.5) = 13.32\%$ and $D^{PI^{0.65}}(m = 0.5) = 11.53\%$

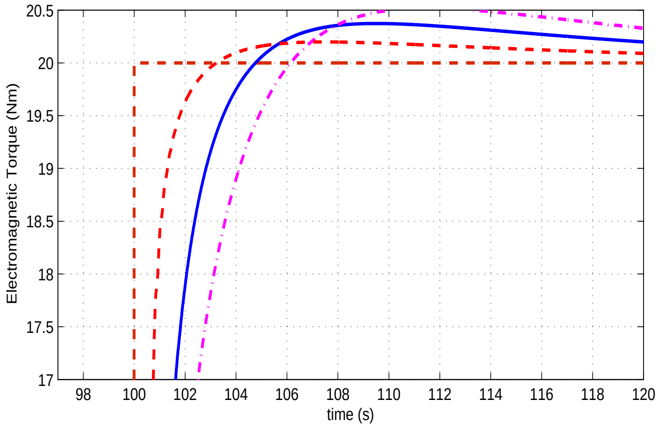

Figure 9: Evolution of the electromagnetic torque for different values of process gain K, (- $K = K_{nom}$; -.- $K = 150\% K_{nom}$; - - $K = 50\% K_{nom}$) $(PI^{0.92})$.

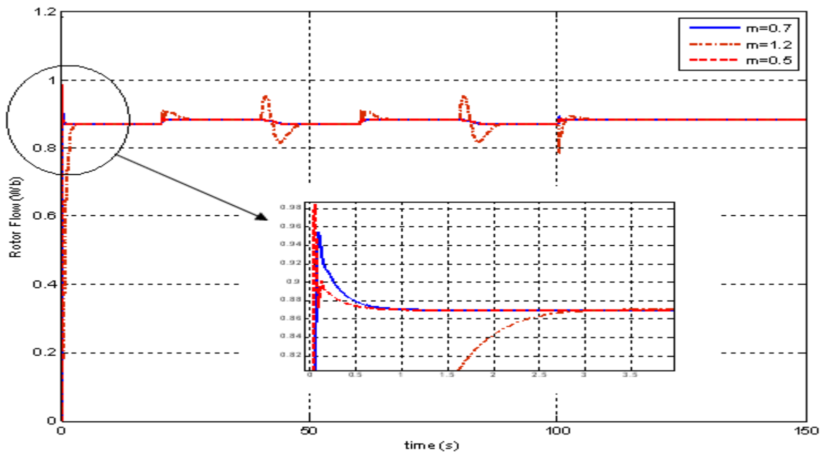

Figure 10: Response flux for different values of the damping factor m (conventional PI).

6 Conclusion

The nonlinear control system with a fractional order controller was presented in this paper, with a comparative study of the conventional controller. We define the correction order and fractional approximation of Charef to determine the rational expression of the integration and the derivation of the correction. The adjustment of the order of fractional order (α, β) is done by minimizing the control error defined by ISE using the Hall-Sartoruis method. The results obtained by simulation and comparative study demonstrate the performance of the control technique with fractional order correction in the presence of load variation and control parameters, as well as the profitability of ISE using the method of Hall-Sartoruis.

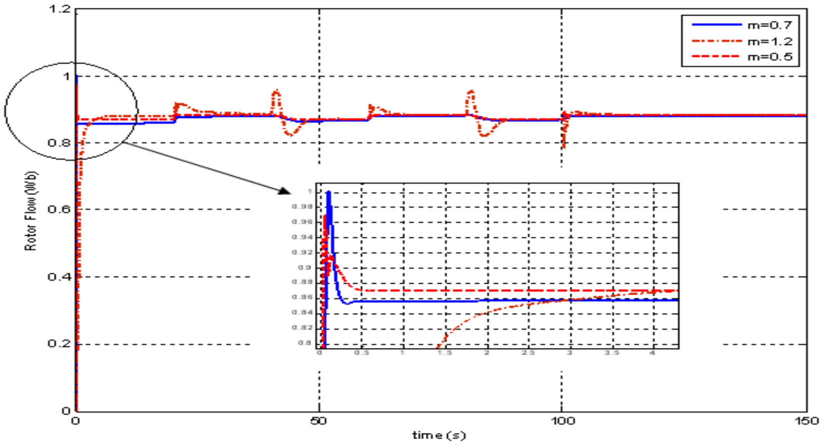

Figure 11: Response flux for different values of the damping factor m ($PI^{0.65}$).

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