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Analytical and Experimental Investigation of Vertical Vibration of a Freight Wagon in the Presence of Mechanical Asymmetry

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Abstract: Production and construction asymmetry of railway vehicles in the presence of multiple track irregularities on the rail influences the time flow of the wheel. It has an influence on wheel and rail wear defects, especially on driving safety. Production and construction asymmetry was found during the experimental investigation of the basic parameters of mechanical properties of a double-axel freight wagon of Smmps type. This was an impulse for a systematic investigation of the influence of production and construction asymmetry on the vertical dynamic of complex mechanical systems, such as a railway vehicle. The current contribution introduces a methodology of analytical solution of the influence of production and construction asymmetry on the vertical dynamic response of a double-axel freight wagon in the presence of multiple track irregularities. Measured field data were used to validate the model.

Keywords: *asymmetry; analytical model; experiment; vertical vibration.*

Mathematics Subject Classification (2010): 93A10.

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1 Introduction

This paper was successfully presented at the conference of Dynamic Systems Theory in Lodz, Poland, 2013. In the present paper, an analytical method to study the effects of production and construction asymmetry on the vertical vibration response of a railway wagon in the presence of multiple track irregularities and their consequences is presented. A review of published material on the subject revealed that many analyses [18] have been made from which general conclusions regarding the dynamic behavior of complex mechanical systems were drawn. The fundamentals of this subject are covered in detail in [3], and will not be treated here. The reader is referred to [4,5,12] for further background.

Investigation of the influence of asymmetry on ground vehicles as a simple model was introduced in 1925 and today it is mostly used as a textbook example in vibration of mechanical system with two degrees of freedom [2, 5]. The full vehicle model dealing with the influence of production and construction asymmetry on the vertical vibration response of a railway wagon in the presence of multiple track irregularities is rarely described in literatures. Therefore, it is the intention of this work to extend preceding analysis by introducing the effect of production and construction asymmetry on the vertical vibration response of complex mechanical systems under multiple track excitations. In this paper a double-axel freight wagon modeled as a 9 DOF three-dimensional system intended for the investigation of the effect of production and construction asymmetry on vertical vibration response in the presence of multiple track irregularities and their consequences is presented.

2 Literature Review

In analyzing the interaction between the train and the track, the vehicle system can be modeled as one-dimensional, two-dimensional, or three-dimensional model. The simplest vehicle model is a single degree of freedom (DOF) one-dimensional model, which considers a single wheel with static force representing the static load due to the car-body and bogie where the contact between the wheel and rail is maintained by either linear or non-linear spring. This model has been applied in a number of published studies concerned with dynamic wheel-rail interaction; see for example [9]. The single DOF is considered sufficient for high frequency vibration analysis considering the interaction between the wheel and rail with surface irregularities. However, this model is insufficient to analyze the contributions due to pitch and roll motions of the vehicle on wheel-rail impact load or to investigate the effect of multiple defects in different wheel-sets and/or multiple surface irregularities on the rail.

Alternatively, two-dimensional models that include half of the car-body and two bogies and four wheel-sets have been most widely formulated and applied for studies on wheel-rail interactions. *Nielsen and Igeland* [17], developed a four-DOF two-dimensional pitch plane model in order to study the influence of wheel and rail imperfections on vehicle-track interaction. This model has been further employed by *Dong* [15] and *Cai* [16] in order to simulate the vehicle-track interaction under wheel defects. Several two-dimensional vehicle models have also been formulated with 10-14 DOF that consist of half bogie and a quarter of the car-body weight and include the pitch motion of both the car-body and bogie [13, 14]. Such a model would be sufficient to analyze the dynamic interaction between the leading and trailing bogie and wheels and effect of the cross wheel defects. However, contributions due to either pitch or roll motion of the car-body and bogies have to be neglected in such models.

Three-dimensional vehicle models have been developed incorporating a full or half of the car-body, two bogies and two wheel-sets. Such models permit dynamic coupling between the leading and trailing bogies. *Sun et al.* [10] developed a comprehensive three-dimensional vehicle model in order to study lateral and vertical dynamics of the wagon-track system [11]. Such a model provides all the advantages of roll, pitch plane models and is quite adequate for the investigation of the influences of coupled vertical, the pitch and lateral dynamics of the vehicle [7]. The pitch and roll motions of the car-body and bogie that could enhance the wheel-rail impact force caused by the wheel and rail irregularities can be adequately investigated. However, the effects of production and construction asymmetry on the vertical vibration response in the presence of multiple track irregularities and their consequences have never been investigated with this full three-dimensional vehicle model. Therefore it is the intension of this paper to introduce an analytical method to solve vertical vibration response of complex mechanical system with multi-DOF. This method is limited to vertical vibration responses only.

3 Railway Vehicle Model

In the present paper an analytical model of a railway vehicle was developed as illustrated in Figure 1. The model consists of a car body, two bogie frames and four wheel-sets. The car body is modeled as a rigid body having a mass m, and having moment of inertia J_x and J_y about the transverse and longitudinal centroidal horizontal axes, respectively. While the bogie frames are considered as rigid bodies with m_1 and m_2, with moment of inertia J_{x1} and J_{y1} for the front bogie and similarly rear bogie having moment of inertia J_{x2} and J_{y2} about the transverse and longitudinal centroidal horizontal axes, respectively. The springs in the primary and secondary suspension system are characterized by spring stiffness constant k_{jki} and damping coefficient b_{jki}, where $j = 1, 2$, quadrant $k = 1, 2, 3, 4$ and spring position orders $i = 1, ..., n$. Assuming small vertical motion and the vehicle car body to be rigid, its motion may be described by the relative vertical displacement w_t and rotations about the main longitudinal horizontal axis φ_x and about main the transverse horizontal axis φ_y. Likewise, the motions of the two bogie frames are described by w_1, φ_{x1}, φ_{y2} for the front bogie frame and w_2, φ_{x2}, φ_{y2} for the rear bogie frame each about their centroidal. The railway wagon is thus represented by a 9 DOF mechanical system.

4 Analytical Method and Solution

A number of analytical solutions to vehicle dynamic response have been developed in the past. Some authors have considered the mechanical analog of the DNA base pair oscillations to analysis rotational oscillations of a DNA fragment in detail, see [8]. To facilitate analyse, it is essential to reduce the complex vehicle vibrating system to its simplest elements. At the same time, careful judgment is called for to avoid assumptions that are not in accord with the basic realities of the situation. Hence with this point in mind, in the present paper the analytical solution of the railway vehicle is considered to be a system of three rigid bodies with 9 degrees of freedom coupled by spring-damper elements with the consideration of linear viscous damping, as shown in Figure 2.

The equations of motion for the railway model considered in this paper are derived from the Lagrange equation of motion and therefore it is necessary to determine the

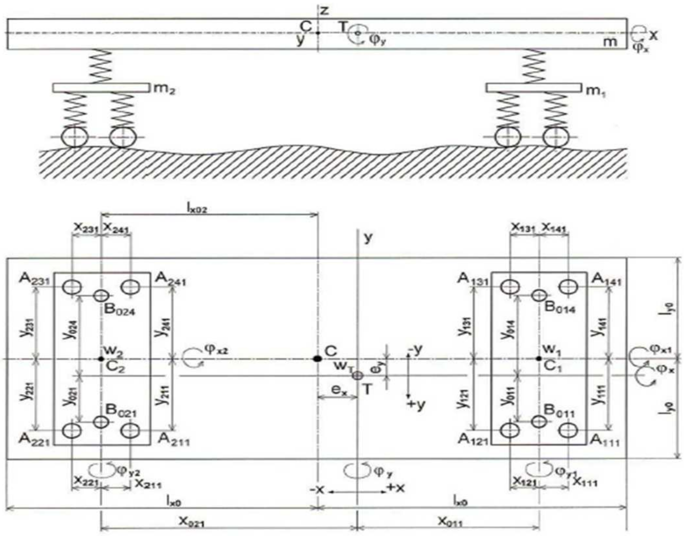


Figure 1: The analytical model.

kinetic energy E_k, potential energy E_p and Rayleigh dissipation function R_d of the mechanical system. The vectors of generalized coordinates system are given as

$$q_j(t) = [w, w_1, w_2, \varphi_x, \varphi_y, \varphi_{x1}, \varphi_{y1}, \varphi_{x2}, \varphi_{y2}]^T, \dot{q}_j(t), \ddot{q}_j(t). \quad (1)$$

The kinetic energy:

$$E_k = \frac{1}{2}m\dot{w}^2 + \frac{1}{2}\left(J_x\dot{\varphi_x}^2 + J_y\dot{\varphi_y}^2 - 2D_{xy}\dot{\varphi_x}\dot{\varphi_y}\right) + E_{k1} + E_{k2}, \quad (2)$$

where

$$E_{k1} = \frac{1}{2}m_1\dot{w_1}^2 + \frac{1}{2}J_{x1}\dot{\varphi_{x1}}^2 + \frac{1}{2}J_{y1}\dot{\varphi_{y1}}^2, \quad E_{k2} = \frac{1}{2}m_2\dot{w_2}^2 + \frac{1}{2}J_{x2}\dot{\varphi_{x2}}^2 + \frac{1}{2}J_{y2}\dot{\varphi_{y2}}^2.$$

To determine the potential energy of the mechanical system, it is necessary to determine the displacements, marked by A_{jki}, of the individual springs in the primary and secondary suspension system w_{jki} characterized by spring stiffness constant k_{jki}, where $j = 1, 2$, quadrant $k = 1, 2, 3, 4$ and spring position orders $i = 1, ..., n$.

Points	Vertical displacements	Constant stiffness
$A111$	$w_{111} = w_1(t) - y_{111}\varphi_{x1}(t) + x_{111}\varphi_{y1}(t) - h_{111}$	k_{111}
$A121$	$w_{121} = w_1(t) - y_{121}\varphi_{x1}(t) + x_{121}\varphi_{y1}(t) - h_{121}$	k_{121}
$A131$	$w_{131} = w_1(t) - y_{131}\varphi_{x1}(t) + x_{131}\varphi_{y1}(t) - h_{131}$	k_{131}
$A141$	$w_{141} = w_1(t) - y_{141}\varphi_{x1}(t) + x_{141}\varphi_{y1}(t) - h_{141}$	k_{141}

Table 1: Front bogie (m_1) for $j = 1$.

Points	Vertical displacements	Constant stiffness
$A211$	$w_{211} = w_2(t) - y_{211}\varphi_{x2}(t) + x_{211}\varphi_{y2}(t) - h_{211}$	k_{211}
$A221$	$w_{221} = w_2(t) - y_{221}\varphi_{x2}(t) + x_{221}\varphi_{y2}(t) - h_{221}$	k_{221}
$A231$	$w_{231} = w_2(t) - y_{231}\varphi_{x2}(t) + x_{231}\varphi_{y2}(t) - h_{231}$	k_{231}
$A241$	$w_{241} = w_2(t) - y_{241}\varphi_{x2}(t) + x_{241}\varphi_{y2}(t) - h_{241}$	k_{241}

Table 2: Rear bogie (m_2) for $j = 2$.

In case of the car body (m), with the point marked by B_{jki}, their coordinates x_{jki}, y_{jki}, $j = 0$, $k = 1$, 2 respectively, $i = 1$, 4 and the k_{jki} individual springs as shown in Figure 2, are:

Points	Vertical displacements	Const. stiff.
$B011$	$w_{011} = w(t) - y_{011}\varphi_x(t) + x_{011}\varphi_y(t) - w_1 + (y_{011} + e_y)\varphi_{x1}$	k_{011}
$B014$	$w_{014} = w(t) - y_{014}\varphi_x(t) + x_{014}\varphi_y(t) - w_1 + (y_{014} + e_y)\varphi_{x1}$	k_{014}
$B021$	$w_{021} = w(t) - y_{021}\varphi_x(t) + x_{021}\varphi_y(t) - w_2 + (y_{021} + e_y)\varphi_{x2}$	k_{021}
$B024$	$w_{024} = w(t) - y_{024}\varphi_x(t) + x_{024}\varphi_y(t) - w_2 + (y_{024} + e_y)\varphi_{x2}$	k_{024}

Table 3: Car body (m).

The potential energy is:

$$E_p = \frac{1}{2}\sum_{j=1}^{2}\sum_{k=1}^{4}\sum_{i=1}^{k_j} k_{jki} w_{jki}^2 + \frac{1}{2}\sum_{j=0}\sum_{k=1,2}\sum_{i=1,4} k_{jki} w_{jki}^2. \tag{3}$$

The Rayleigh dissipation function is:

$$R_d = \frac{1}{2}\sum_{j=1}^{2}\sum_{k=1}^{4}\sum_{i=1}^{k_j} b_{jki} \dot{w}_{jki} + \frac{1}{2}\sum_{j=0}\sum_{k=1,2}\sum_{i=1,4} b_{jki} w_{jki}. \tag{4}$$

The equations of motion for the railway model considered in this paper are derived from the Lagrange equation of motion (5). By substituting equations (2), (3) and (4) into equation (5) and after the derivation of equation (5) we have

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_j}\right) - \frac{\partial E_k}{\partial q_j} + \frac{\partial E_p}{\partial q_j} + \frac{\partial R_d}{\partial \dot{q}_j} = Q_j \tag{5}$$

for j=1, ..., p=9. According to [1] and [4], in the time domain the equations of motion for this system may be obtained in the general form as

$$\mathbf{M}\ddot{q}(t) + \mathbf{B}\dot{q}(t) + \mathbf{K}q(t) = \mathbf{Q}_j(t), \tag{6}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{B}$ is the damping matrix, $\mathbf{K}$ is the stiffness matrix and for generalization, all elements of matrix $\mathbf{B}$ and $\mathbf{K}$ are considered not to equal zero.

$$q_j(t) = [w, w_1, w_2, \varphi_x, \varphi_y, \varphi_{x1}, \varphi_{y1}, \varphi_{x2}, \varphi_{y2}]^T, \dot{q}_j(t), \ddot{q}_j(t)$$

are the vectors of the generalized coordinates and $\mathbf{Q}_j(t)$ is the vector of the generalized kinematic excitation functions. After dividing equation (6) by the respective diagonal element of the mass matrix $\mathbf{M}$ and after the Laplace transformation for the zero initial conditions $q(0)$ and $\dot{q}(0)$, the system of differential equations is transformed to the system of algebraic equations

$$\mathbf{G}\bar{q}(s) = \bar{f}(s), \tag{7}$$

where s is the parameter of transformation, $\bar{q}(s)$ and $\bar{f}(s)$ are the vectors of the generalized coordinates $q(t)$ and forces $f(t)$. It holds for the elements of the matrix $\mathbf{G}$:

$$\begin{array}{rc} g_{ij} = s^2 + \beta_{ij}s + \kappa_{ij}, & \text{for } i = j, \\ g_{ij} = -\delta_{ij}s^2 + \beta_{ij}s + \kappa_{ij}, & \text{for } i \neq j \text{ and for } i = 4,6,8 \text{ and } j = i+1, \\ g_{ij} = \beta_{ij}s + \kappa_{ij}, & \text{for } i \neq j \text{ and for } i = 5,7,9 \\ & \text{and } j = i-1 \text{ for } i = 3,...,9 \text{ and } j = 1,...,9,, \end{array}$$

where $\delta_{12} = -\frac{D_{xy}}{J_x}$ and $\delta_{21} = -\frac{D_{xy}}{J_y}$ are the elements representing the influence of asymmetric distribution of the sprung mass.

For solving the system of algebraic equations (7), it is possible due to small number of equations, to apply the Cramer rule [1] as follows

$$\bar{q}_j(s) = \sum_{i=1}^{n/2} (-1)^{j+i} \bar{f}_i(s) \frac{D_{ji}(s)}{D(s)}, \quad j = 1,2,...,n/2, \quad n = 18. \tag{8}$$

This method is suitable regarding the process of obtaining the vector of the generalized coordinates $q(t)$ by the inverse transformation. In order to determine the original $q_j(t)$ of the corresponding image $\bar{q}_j(s)$ it is necessary to transform equation (8) to the form of convolution integral. Therefore, it is necessary to find the poles of the characteristic polynomial $D(s)$ of equation (8) [1]. The poles are supposed to be in the form of complex conjugates $s_i = -Res_k \pm Ims_k$, for k= 1,2,..., $n/2$. To evaluate the poles of the characteristic equation $D(s)$, it is necessary to equate the polynomial in the form of the product of the quadratic polynomials using the product of the roots factors $s^2 + p_k s + r_r$, for k= 1,2,..., $n/2$. The polynomial responding to the sub-determinant $D_{ji}(s)$ is determined using the same algorithm.

In order to determine the original $q_j(t)$ of the corresponding image $\bar{q}_j(s)$ it is suitable to transform equation (8) to the form of convolution. Therefore it is possible to transfer the ratio of the determinants in equation (8) to the sum of partial fractions in the form

$$\frac{D_{ij}(s)}{D(s)} = \frac{\sum\limits_{r=1}^{n/2} \left[(K_{ji,r}s + L_{ji,r}) \prod\limits_{k=1,k\neq i}^{n/2} \left(s^2 + p_k s + r_k\right) \right]}{\prod\limits_{k=1}^{n/2} \left(s^2 + p_k s + r_k\right)} = \sum_{r=1}^{n/2} \frac{K_{ji,r}s + L_{ji,r}}{s^2 + p_k s + r_k}, \tag{9}$$

where the constants $K_{ji,r}$ and $L_{ji,r}$ for j= 1,2,..., $n/2$, i= 1,2,..., $n/2$, r= 1,2,..., $n/2$, can be determine from the condition of the coefficients equality of the identical powers of the parameter s in the numerator of the fractional equation (9). By substituting equation (9) into equation (8) for the determination of the image of the generalized coordinates $\bar{q}(s)$, for j= 1,2,..., $n/2$, the latter can be modified as follows

$$\bar{q}_j(s) = \sum_{i=1}^{n/2} (-1)^{j+i} \bar{f}_i(s) \sum_{k=1}^{n/2} \frac{K_{ji,k}s + L_{ji,k}}{s^2 + p_k s + r_k}. \tag{10}$$

After inverse transformation of equation (10) for the function of the generalized coordinate $q_j(t)$, for $j = 1, 2, 3, \ldots, 9$, the form of the sum of convolution integrals is obtained as follows

$$q_j(t) = \sum_{i=1}^{9} (-1)^{j+i} \sum_{k=1}^{9} \left[G_1 + G_2\right], \tag{11}$$

where

$$G_1 = K_{ji,k} \int_0^t F_i(\tau)\, e^{-\beta_k (t-\tau)} cos\left[\Omega_k (t-\tau)\right] d\tau,$$

$$G_2 = \frac{L_{ji,k} - \beta_k K_{ji,k}}{\Omega_k} \int_0^t F_i(\tau)\, e^{-\beta_k (t-\tau)} sin\left[\Omega_k (t-\tau)\right] d\tau.$$

Equation (11) shows the solution for a linear viscous damped mechanical system, where j-th component of vector of generalized coordinates $q_j(t)$ is the sum of convolution integrals, multiplied by i-th generalized kinematic excitation elements $F_i(t)$ designated by the product of spring constant and height of the road or rail surface unevenness and by product of damping coefficient b_{jik}, and time derivative of height contact place of the m-index wheel at specific crossing velocity, to the k-th harmonic component with its own natural frequency Ω_k. K_{jik} and L_{jik} are unknown coefficients of amplitude, depending on the mechanical properties of the system under consideration. Vector components of the kinematic excitation function $F_i(t)$ are given in the range of $0 \leq t$.

5 Production and Construction Asymmetry of the Mechanical System

In this paper asymmetry of the distributed sprung mass of the railway vehicle is simulated as shown in Figure 2, where Figure 2a shows the symmetrical case, where $T = C$, $e_x = e_y = 0$ and Figure 2b shows the asymmetrical case, where $T \neq C$, $e_x \neq 0$ and $e_y \neq 0$. The position of the external weight placed on the surface of the wagon introduces weight eccentricities due to the uneven distribution of the sprung mass of the mechanical system. This effect causes the center of mass to be arbitrarily positioned so that no symmetry exists in the system, as a result of this the system center of mass T is shifted along the x-axis and y-axis directions with respect to the system's geometrical center. Meanwhile, Figure 2 shows the arrangement of rail defects as kinematic excitations; Figure 2c shows the symmetrical arrangement of the multiple track irregularities on the track, while Figure 2d shows the asymmetrical arrangement of rail defects. Meanwhile Figure 3 shows different cases of asymmetry and the arrangement of the multiple track irregularities (kinematic excitation) on both rails. In this paper the multiple track irregularities are modeled as a unit step function.

6 Experimental Tests

Experimental tests were done on a four-axel freight wagon of *Smmps* type. The railway freight wagon was modified in a way to be in accordance with the requirements of the analytical model derivate in Section 2. The original bogies were removed from the wagon and replaced by another bogie of Y25 type from a passenger freight car. The outer springs of the primary spring system of these bogie frames were removed and secondary spring system, consisting of three springs was fitted to the bogie frames. The bogie frames were

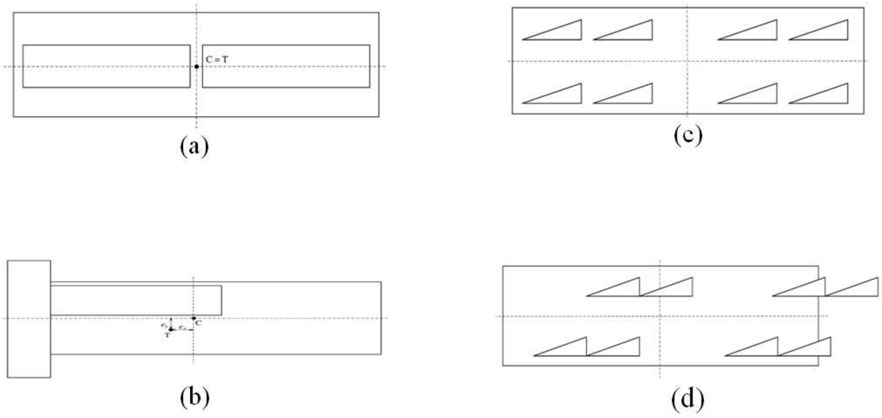


Figure 2: The analytical - symmetry and asymmetrical model, (a) Symmetrical and (b) asymmetrical distribution of the sprung mass (c) Symmetrical and (d) asymmetrical of multiple track irregularities.

Chocks / p. of loads	I	II	III	IV	V	VI
A	A - I	A – II	A – II	A – IV	A – V	A – VI
B	B - I	B – II	B – III	B – IV	B – V	B – VI
C	C - I	C – II	C – II	C – IV	C – V	C – VI
D	D - I	D – II	D – II	D – IV	D – V	D – VI
E	E - I	E – II	E – III	E – IV	E – V	E – VI

Figure 3: Symmetric and asymmetric model.

fixed to the wagon by means of wire ropes net to keep the car body in equilibrium, see Figure 4 and Figure 5 respectively.

Figure 4: Secondary spring system.

Figure 5: Connection of bogie frame to car body.

6.1 Test procedures

The vertical vibration responses of the freight wagon were done as follows:

- The freight wagon loaded as illustrated in Table 1 was towed by a shunting locomotive to move over track irregularities. The locomotive was connected to the car body with a wire rope. The wedge blocks stimulated the Heaviside unit function.
- The measurements were sensed by means of HBM amplifiers, signals went through the low-pass filter 32 kHz into the digital-to-analogue system DAS 48.
- Each test was repeated 2 – 3 times. A total of 89 tests were measured. Twenty quantities were measured during the tests – the bogie frames relative vertical vibration response with respect to the axel-box (9 sensors), car body relative vertical vibration response with respect to the front and the rear bogie frames (4 sensors), vertical acceleration of the car body (5 sensors) and finally, acceleration of the bogie frames (2 sensors). The purpose of these tests was to determine and record time histories of relative vertical vibration response of the freight wagon in the presences of production and construction asymmetry and multiple general kinematic excitations to verify the theoretical model in Section 2.

7 Analytical Solutions

Two types of analysis were performed in this paper in order to investigate the effects of production and construction asymmetry on the vertical vibration response of the railway vehicle in the presence of multiple track irregularities. The first analysis was done for simulated analytical data set and the second was for experimental data to validate the analytical model. The analytical data was processed using a MatLab code, which was written specifically for this investigation. In regard of large amount of collected data, it was not possible to process and include all the results into this one investigation report, because of its very limited extent. Therefore, the present work comprises only general conclusions. Figure 6, shows the analytical results processed using a MatLab code for the symmetrical (a) and asymmetrical (b) distribution of the sprung mass running over uneven track irregularities (d), as shown in Figure 2. The results showed the expected trends of vertical vibration response of the model with no definite cut off between stable and unstable behavior.

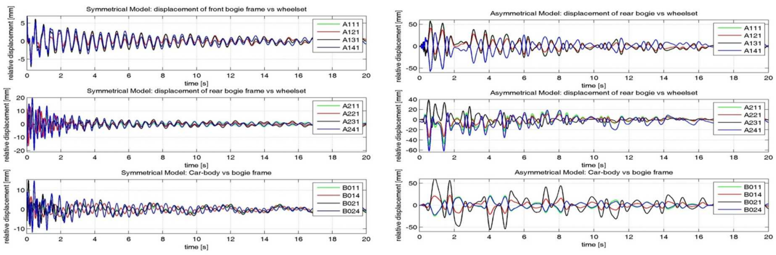
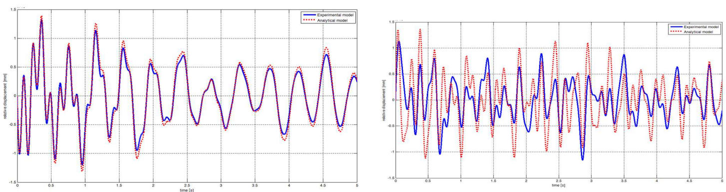


Figure 6: Vertical dynamic response due to a single unit step on all wheels on the right side of the wagon.

8 Validation of the Developed Model

The developed model has been validated using the experimental data reported by *J.Soukup* and *J.Volek*, see [1] for more detail. The parameters employed in the simulation are obtained from reported studies [6]. The comparisons between the responses obtained by the developed model in Section 4 with that of the reported study are shown in Figure 7. The results showed expected trends of vertical vibration response in the presence of production and construction asymmetry of the mechanical system. It is quite evident, that good agreement with a 9DOF vehicle model has been achieved, but several limitations of the model have been identified. The significance of these limitations is currently being investigated with more additional degrees of freedom. It can be concluded that, the influence of production and construction asymmetry on the vertical vibration response in the presence of multiple track irregularities is obvious. Hence detail analysis of this phenomenon is a necessity in railway vehicle design process.



(a) Symmetrical model (b) Asymmetrical model

Figure 7: The comparisons between the responses obtained by the developed model with that of the reported study.

9 Conclusion

The present paper confirmed the influence of production and construction asymmetry of railway vehicle in the presence of multiple track irregularities inputs. In regard to the large amount of collected data, it was not possible to process and include all the results into this one investigation report, because of its very limited extent. Therefore, the present work comprises only general conclusions.

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