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The Duffing–Van der Pol Equation: Metamorphoses of Resonance Curves

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Abstract: We study dynamics of the Duffing–Van der Pol driven oscillator. Periodic steady-state solutions of the corresponding equation are determined within the Krylov-Bogoliubov-Mitropolsky approach to yield dependence of amplitude on forcing frequency as an implicit function, referred to as resonance curve or amplitude profile. Equations for singular points of resonance curves are solved exactly. We investigate metamorphoses of the computed amplitude profiles induced by changes of control parameters near singular points of these curves since qualitative changes of dynamics occur in neighbourhoods of singular points. More exactly, conditions for birth of resonances as well as for attractor crises are found. Bifurcation diagrams are computed to show good agreement with theoretical analysis.

Keywords: *oscillators; resonance curves; singular points.*

Mathematics Subject Classification (2010): 34C15, 70K30, 37G10.

1 Introduction

Nonlinear oscillators have many important applications in various areas of science and engineering [1, 2]. In this paper we investigate Duffing–Van der Pol oscillator which has been extensively studied due to potential applications in physics, chemistry, biology, engineering, electronics, and many other fields, see [3, 4] and references therein.

The periodically forced Duffing – Van der Pol oscillator (DvdP) is written as:

$$\frac{d^2x}{dt^2} - \left(b - cx^2\right)\frac{dx}{dt} + ax + dx^3 = f\cos\omega t. \tag{1}$$

There are three main cases of the Duffing potential $V(x) = \frac{1}{2}ax^2 + \frac{1}{4}dx^4$: (i) single well ($a > 0$, $d > 0$), (ii) double well ($a < 0$, $d > 0$), and (iii) double hump ($a > 0$, $d < 0$).

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