Pullback Attractors of Nonautonomous Boundary Cauchy Problems

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Abstract: In this work, we establish the existence of pullback attractors for nonautonomous nonlinear boundary Cauchy problems. We apply our result to a reaction-diffusion equation.

Keywords: nonautonomous boundary Cauchy problem; pullback attractors; reaction-diffusion equation.


1 Introduction

Consider the nonlinear boundary Cauchy problem for arbitrary \( s \in \mathbb{R} \)

\[
\begin{align*}
\frac{d}{dt} u(t) &= A_{\text{max}}(t) u(t), \quad t \in [s, \infty),
L(t) u(t) &= f(t, u(t)), \quad t \in [s, \infty),
\end{align*}
\]

\[u(s) = x,\]

(1)

where \( A_{\text{max}}(t) \) is a closed operator on a Banach space \( X \) endowed with a maximal domain \( D(A_{\text{max}}(t)) \), and \( L(t) : D(A_{\text{max}}(t)) \to \partial X \), with a ‘boundary space’ \( \partial X \) and a function \( f : \mathbb{R} \times X \to \partial X \), the solution \( u : [s, \infty) \to X \) takes the initial value \( x \in X \) at time \( s \). Moreover, the restriction \( A(t) := A_{\text{max}}(t)|_{\ker(L(t))} \) is assumed to generate an evolution family \( (U(t,s))_{t \geq s} \), on the state space \( X \). That is \( U(t,s)x \) is a solution of the corresponding linear boundary Cauchy problem of (1) given by

\[
\begin{align*}
\frac{d}{dt} u(t) &= A_{\text{max}}(t) u(t), \quad t \in [s, \infty),
L(t) u(t) &= 0, \quad t \in [s, \infty),
\end{align*}
\]

\[u(s) = x.\]

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