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Extremal Mild Solutions for Finite Delay Differential Equations of Fractional Order in Banach Spaces

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Abstract: In this paper, we study the existence and uniqueness of extremal mild solutions for finite delay differential equations of fractional order in Banach spaces with the help of the monotone iterative technique based on lower and upper solutions. This technique uses the iterative procedure starting from a pair of ordered lower and upper solutions to obtain the extremal mild solutions. We also use the theory of fractional calculus, semigroup theory and measures of noncompactness to obtain the results. An example is presented to illustrate the main result.

Keywords: *fractional delay differential equations; semigroup theory; monotone iterative technique; Kuratowskii measures of noncompactness.*

Mathematics Subject Classification (2010): 34A08, 34G20, 34K30.

1 Introduction

In this paper, our aim is to study the existence of extremal mild solutions for the following finite delay differential equations of fractional order in an ordered Banach space X of the form:

$$\begin{cases} {}^cD^\alpha x(t) &=& Ax(t) + f(t, x_t), \quad t \in J = [0, b], \\ x_0(\nu) &=& \phi(\nu), \quad \nu \in [-a, 0], \end{cases} \tag{1}$$

where state $x(.)$ takes value in the Banach space X endowed with norm $\|.\|$; ${}^cD^\alpha$ is the Caputo fractional derivative of order α, $0 < \alpha < 1$; $A : D(A) \subset X \to X$ is a closed linear densely defined operator; A is an infinitesimal generator of a strongly continuous semigroup $\{T(t)\}_{t\geq 0}$ on X. The function $f : J \times D \to X$ is given nonlinear function, here $D = C([-a, 0], X)$. If $x : [-a, b] \to X$ is a continuous function, then x_t denotes the

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