Integrable Time-Dependent Dynamical Systems: Generalized Ermakov-Pinney and Emden-Fowler Equations

Partha Guha\textsuperscript{1*} and Anindya Ghose Choudhury\textsuperscript{2}

\textsuperscript{1} S.N. Bose National Centre for Basic Sciences, JD Block, Sector III, Salt Lake, Kolkata-700098, India.
\textsuperscript{2} Department of Physics, Surendranath College, 24/2 Mahatma Gandhi Road, Calcutta-700009, India

Received: January 23, 2014; Revised: October 17, 2014

Abstract: We consider the integrable time-dependent classical dynamics studied by Bartuccelli and Gentile (Phys Letts. A\textbf{307} (2003) 274–280; Appl. Math. Lett. \textbf{26} (2013) 1026–1030) and show its power to compute the first integrals of the (generalized) Ermakov-Pinney systems. A two component generalization of the Bartuccelli-Gentile equation is also given and its connection to Ermakov-Ray-Reid system and coupled Milne-Pinney equation has been illucidated. Finally, we demonstrate its application in other integrable ODEs, in particular, using the spirit of Bartuccelli-Gentile algorithm we compute the first integrals of the Emden-Fowler and describe the Lane-Emden type equations. A number of examples are given to illustrate the procedure.

Keywords: time-dependent harmonic oscillator; Ermakov-Pinney equation; first integrals; Ermakov-Lewis invariant; Emden-Fowler equation.

Mathematics Subject Classification (2010): 34A05, 34A34, 34C14.

* Corresponding author: mailto:partha@bose.res.in