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Stabilizing Sliding Mode Control for Homogeneous Bilinear Systems

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Abstract: The stabilization of homogeneous bilinear systems constitutes the main interest of this paper. A sliding mode control is suggested and a stability study is held leading to sufficient conditions of global stabilization. The sliding surface is determined through the resolution of the nonlinear constraints of stabilization. Simulations on numerical examples are presented proving the effectiveness of the proposed approach.

Keywords: *homogeneous bilinear systems; sliding mode control; stabilization.*

Mathematics Subject Classification (2010): 11D09, 93D20, 39B62.

1 Introduction

Bilinear systems constitute an important class of nonlinear systems. Since their introduction in the early sixties, they have got great interest and have been used to model processes in several fields; biologic, ecologic, economic, social ... [4, 16, 17]. As they are partially linear in state and in input without being jointly linear in both, they constitute a gateway between linear and nonlinear systems and that's why they need special attention in their study. In the last decades, many researchers investigated the control design and the stability analysis of this special category of systems [1, 9, 13, 15, 19, 20].

Many results in this field are yet demonstrated, since the stabilization by linear or quadratic state feedback has been widely treated especially for non homogeneous bilinear systems. However it was shown that there exists a large class of homogeneous bilinear systems which can not be stabilized by a continuous feedback even in planar case [6]. In fact for this type of systems the relative degree isn't defined in zero and the linearized

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system is control independent. For such systems, R. Chabour and al. proposed, in the case of second order dimension, zero degree homogeneous positive controls. For three dimensional systems, Celikovsky investigated in [5] the global asymptotic stabilization by constant feedback and the practical stabilization by a family of linear feedbacks for a special class of single input homogeneous bilinear system ($\dot{x} = Ax + uNx$), where A is a diagonal matrix with a negative trace and N is a skew-symmetric matrix. This work was extended by O. Chabour for n dimensional systems where matrix A has not to be diagonal and its trace has not to be negative, [7]. An integrated overview of bilinear system research presented by Mohler and al. in [18] deals with the efficiency of the optimal control and the variable structure control such as bang-bang control. Later, in [2] Amato and al. suggested a procedure to design a stabilizing state feedback controller formulated in a convex optimisation problem involving LMIs.

In this paper, we interest in the stabilization of homogenous bilinear systems of any dimension. No restriction on the system's structure are imposed. The sliding mode approach is adopted to design a variable structure control. Stability study is investigated leading to sufficient conditions of global stabilization formulated in computationally resolvable nonlinear matrix constraints. Besides a simplified algorithm is provided making use of the linear quadratic control approach. The resolution of the stabilization constraints system enables to provide the matrix C characterising the sliding surface ($S = Cx$). The proposed approach is successfully applied to homogeneous bilinear systems of different orders.

In the following section the control design procedure is detailed for homogeneous bilinear systems leading to the definition of two control laws; the switching control needed in the reaching phase toward the sliding surface, and the equivalent control required while the system slides on the surface. In Section 4 an extended stabilization study is carried out based on quadratic Lyapunov function. To formulate the global stabilization conditions during the sliding mode in resolvable matrix constraints the "vec" operator and the tensor product are employed. Finally two numeric examples are considered in Section 6 to underline the effectiveness of the proposed approach.

2 Homogeneous Bilinear Systems and Sliding Mode Control Design

Bilinear systems are generally represented by a state equation of the form:

$$\dot{x} = Ax(t) + Bu(t) + \sum_{j=1}^{m} N_j x(t) u_j(t), \tag{1}$$

where $x \in X \subset \Re^n$ is the state vector, $u = [u_1 \dots u_m]^T \in U \subset \Re^m$ is the control input, A, B and N_j, $j = 1 \dots m$, are matrices of suitable dimensions.

When the matrix B is not null, this general form characterises non-homogeneous bilinear systems, and if B is null, the represented system is said to be homogeneous.

As we are interested in this paper in this last class of systems, we will consider the state space equations of the form:

$$\dot{x} = Ax(t) + \sum_{j=1}^{m} N_j x(t) u_j(t). \tag{2}$$

The sliding mode approach consists in bringing the system's state up to a well defined surface where it will slide toward the equilibrium point. Thus the sliding mode control is usually constituted by two parts, the switching control and the equivalent control. The first is discontinuous and it is needed during the reaching phase until the system's state attend the sliding surface, and the second is continuous and aims to keep the state on this surface while sliding.

2.1 Reaching condition and switching control

Let define the sliding surface with $C \in \Re^{p\times n}$:

$$S(t) = Cx(t) = 0. \tag{3}$$

The reaching mode to the sliding surface is guaranteed if [1]

$$\frac{d}{dt}(S^T S) = 2x^T C^T C\dot{x} < 0. \tag{4}$$

When substituting $\dot{x}$ by its expression (2) one gets:

$$2x^T C^T C(Ax + \sum_{i=1}^{m} N_i x u_i) < 0. \tag{5}$$

So the controls u_i $(i = 1...m)$ must be designed such that to satisfy the inequality above. We consider a switching control law defined by:

$$u_{i_s} = \begin{cases} -\alpha \frac{x^T N_i^T C^T Cx|x^T C^T CAx|}{\|x^T C^T CN_i x\|^2}, & if \quad S \neq 0 \; and \; x^T C^T CN_i x \neq 0, \\ 0, & else. \end{cases} \tag{6}$$

Let Ł be the set of the indices i such that $x^T C^T CN_i x \neq 0, \forall x \neq 0$, and let l be the number of its elements, then when substituting u_i by u_{i_s}, the left hand term of the inequality (5) will be reduced to:

$$x^T C^T CAx - \alpha l|x^T C^T CAx| \tag{7}$$

which is negative for all $\alpha > 1$ and $l \geq 1$.

2.2 Sliding mode and equivalent control

In order to keep the system's state on the surface S during the sliding mode, the following condition must be fulfilled:

$$\dot{S} = 0 \quad when \quad S = 0, \tag{8}$$

$$\dot{S} = CAx + \sum_{i=1}^{m} CN_i x u_i. \tag{9}$$

Let Ψ be the set of the indices i such that $CN_i x \neq 0$, $\forall x \neq 0$, and let s be the number of its elements, so we can write

$$\dot{S} = \sum_{i=1}^{s} [\frac{1}{s} CAx + CN_i x u_i]. \tag{10}$$

[1] In the following we will omit the time symbol $'(t)'$ of dynamic variables for the aim of simplification

Thus $\dot{S} = 0$ if we have $u_i = u_{i_{eqv}}$ for all $i = 1, \dots, m$, where

$$u_{i_{eqv}} = \begin{cases} -\frac{1}{s}\frac{(CN_ix)^T CAx}{(CN_ix)^T(CN_ix)}, & if \quad S = 0 \; and \; CN_ix \neq 0, \forall x \neq 0, \\ 0, & else. \end{cases} \tag{11}$$

The homogeneous bilinear system (2) can then be efficiently controlled by the sliding mode control defined by:

$$u = [u_1 \dots u_m]^T \tag{12}$$

where for all $i = 1, \dots, m$

$$u_i = u_{i_s} + u_{i_{eqv}}, \tag{13}$$

The switching control u_{i_s} and the equivalent control $u_{i_{eqv}}$ are those defined by (6) and (11).

3 Stability Analysis

As the considered bilinear system is controlled by the sliding mode control, its behavior depends on two phases: the reaching mode and the sliding mode. The stability of the controlled system is guarantied unless the reaching condition is fulfilled and the system remains stable on the sliding surface. The first condition is already verified ($\frac{d}{dt}(S^TS) < 0$ when $S \neq 0$), so we must prove the stability during the sliding mode.

On the sliding surface the function $S(x) = Cx(t) = 0$ where C is a matrix of dimension $p \times n$. One can write $C = [C_1 \;\; C_2]$ where $C_1 \in \Re^{p\times p}$ and $C_2 \in \Re^{p\times(n-p)}$ then we have: $Cx = C_1x_1 + C_2x_2 = 0$ with $x_1 \in \Re^p$ and $x_2 \in \Re^{n-p}$.

Suppose that [1] $C_1 = I_p$, so we obtain a relationship between x_1 and x_2:

$$x_1 = -C_2x_2. \tag{14}$$

Thanks to the above relationship, the convergence of the system's state to the zero equilibrium point can be demonstrated by the convergence of its second part x_2. Then we can eliminate x_1 from the system and the control formulations. For this consider the following notations:

$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, $N_i = \begin{bmatrix} N_{i_{11}} & N_{i_{12}} \\ N_{i_{21}} & N_{i_{22}} \end{bmatrix}$, $\forall i = 1 \dots m$, with $A_{11}, N_{i_{11}} \in \Re^{p\times p}$,

A_{12}, $N_{i_{12}} \in \Re^{p\times(n-p)}$, A_{21}, $N_{i_{21}} \in \Re^{(n-p)\times p}$, A_{22} and $N_{i_{22}} \in \Re^{(n-p)\times(n-p)}$. So the equation (2) can be detailed as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{i_{11}}x_1 + A_{i_{12}}x_2 \\ A_{i_{21}}x_1 + A_{i_{22}}x_2 \end{bmatrix} + \sum_{j=1}^{m} (\begin{bmatrix} N_{ij_{11}}x_1 + N_{ij_{12}}x_2 \\ N_{ij_{21}}x_1 + N_{ij_{22}}x_2 \end{bmatrix}) u_{i_j}. \tag{15}$$

Replacing x_1 by its expression in (14), the derivative of x_2 can be expressed by:

$$\dot{x}_2 = (A_{22} - A_{21}C_2)x_2 + \sum_{i=1}^{m} (N_{i_{22}} - N_{i_{21}}C_2)x_2u_i. \tag{16}$$

[1] I_p denotes the identity matrix of dimension p

The control u_i on the sliding surface is equal to the equivalent control $u_{i_{eqv}}$ and it can also be expressed as function of x_2:

$$u_i = u_{i_{eqv}} = -\frac{1}{s}\frac{(CN_i x)^T CAx}{(CN_i x)^T (CN_i x)}, \quad \forall i = 1, \dots, s. \tag{17}$$

It is easy to obtain:

$$CN_i x(t) = G_i x_2(t), \quad CAx(t) = Hx_2(t),$$

where

$$\begin{aligned} G_i &= C_2(N_{i_{22}} - N_{i_{21}} C_2) + N_{i_{12}} - N_{i_{11}} C_2, \\ H &= C_2(A_{22} - A_{21}C_2) + A_{12} - A_{11}C_2, \end{aligned}$$

so

$$u_i = u_{i_{eqv}} = -\frac{1}{s}\frac{x_2^T G_i^T H x_2}{x_2^T G_i^T G_i x_2}. \tag{18}$$

Consider the Lyapunov function $V(x_2) = x_2^T P x_2$ where P is a positive definite symmetric matrix, we have to prove that $\dot{V}(x_2) < 0$ for all $x \in X \subset \Re^n$.

$$\dot{V}(x_2) = x_2^T P \dot{x}_2 + \dot{x}_2^T P x_2. \tag{19}$$

Let

$$\mathbb{A} = A_{22} - A_{21}C_2. \tag{20}$$

$$\mathbb{N}_i = N_{i_{22}} - N_{i_{21}}C_2, \ \ \forall i = 1 \dots m. \tag{21}$$

Then the derivative of the Lyapunov function becomes:

$$\dot{V}(x_2) \ = \ x_2^T P[\mathbb{A} - \frac{1}{s}\sum_{i=1}^{s}\frac{x_2^T G_i^T H x_2}{x_2^T G_i^T G_i x_2}\mathbb{N}_i]x_2 \ + \ x_2^T[\mathbb{A} - \frac{1}{s}\sum_{i=1}^{s}\frac{x_2^T G_i^T H x_2}{x_2^T G_i^T G_i x_2}\mathbb{N}_i]^T P x_2. \tag{22}$$

Noting that $\dot{V}(x_2)$ can be rearranged in the following form

$$\dot{V}(x_2) = x_2^T (P\mathbb{A} + \mathbb{A}^T P)x_2 - \frac{1}{s}\sum_{i=1}^{s}\frac{x_2^T G_i^T H x_2}{x_2^T G_i^T G_i x_2} x_2^T (P\mathbb{N}_i + \mathbb{N}_i^T P)x_2 \tag{23}$$

and since the term $x_2^T G_i^T G_i x_2$ is usually positive, then we can deduce that $\dot{V}(x_2) < 0$ if for all $i = 1, \dots, s$ we verify:

$$\begin{cases} x_2^T (P\mathbb{A} + \mathbb{A}^T P)x_2 < 0, \\ x_2^T G_i^T H x_2 x_2^T (P\mathbb{N}_i + \mathbb{N}_i^T P)x_2 \geq 0, \end{cases} \forall \ \ x_2 \neq 0. \tag{24}$$

The first inequality is equivalent to the definite negativity of the matrix $(P\mathbb{A} + \mathbb{A}^T P)$ while the second necessitates additional developments. This latter represents a product of two scalars:

$$(x_2^T \mathbb{V}_i x_2)(x_2^T \mathbb{W}_i x_2), \tag{25}$$

where

$$\begin{cases} \mathbb{V}_i = G_i^T H, \\ \mathbb{W}_i = P\mathbb{N}_i + \mathbb{N}_i^T P. \end{cases} \tag{26}$$

Using the relation between the 'vec' operator and the tensor product ($\otimes$) [3]:

$$vec(AXB) = (B^T \otimes A)vec(X), \tag{27}$$

where A, X and B are any matrices of coherent dimensions, the expression (26) can be reformulated as follows:

$$x_2^T \mathbb{V}_i x_2 x_2^T \mathbb{W}_i x_2 = vec(x_2^T \mathbb{V}_i x_2 x_2^T \mathbb{W}_i x_2) = (x_2^{[2]})^T (\mathbb{W}_i^T \otimes \mathbb{V}_i)(x_2^{[2]}). \tag{28}$$

This expression is strictly positive for all $x_2 \neq 0$ if the matrix $\mathbb{W}_i^T \otimes \mathbb{V}_i$ is positive definite. However, since the vector $x_2^{[2]}$ has redundant terms, it might exist a solution to this problem even with non-positive definite matrix. Therefore, it is possible to relax this condition by eliminating the redundancy in the vector $x_2^{[2]}$. For that a transition matrix T can be introduced, [3],such that:

$$x_2^{[2]} = T\tilde{x}_2^{[2]}. \tag{29}$$

Hence the expression(28) becomes:

$$x_2^T \mathbb{V}_i x_2 x_2^T \mathbb{W}_i x_2 = (\tilde{x}_2^{[2]})^T T^T (\mathbb{W}_i^T \otimes \mathbb{V}_i) T \tilde{x}_2^{[2]}. \tag{30}$$

Finally we can confirm that the derivative of the Lyapunov function (23) is negative definite if we have:

$$\begin{cases} P\mathbb{A} + \mathbb{A}^T P < 0, \\ T^T (\mathbb{W}_i^T \otimes \mathbb{V}_i) T \geq 0, \forall i = 1 \ldots s. \end{cases} \tag{31}$$

The above results are then summarized in the following theorem.

Theorem 3.1 *The homogeneous bilinear system* (2) *is stabilizable by the sliding mode control* (12), (13), (6), (11) *for all real* $\alpha > 1$ *if there exist a positive definite symmetric matrix* P *and a matrix* C_2 *verifying the conditions* (31)*, with all defined notations respected.*

The conditions (31) constitute nonlinear matrix inequalities system which can be solved via a multi-objective optimization function such as 'fgoalattain' or 'fmincon' of the Matlab optimization toolbox. The resolution of this problem will provide the matrix C_2 and the positive definite symmetric matrix P if there exist any.

One way to get round this nonlinear optimization problem is to search C_2 that stabilizes the pair (A_{22}, A_{21}), for example by the linear quadratic regulator function 'lqr' (which ensures the negativity of the first inequality), while verifying the positivity of the second inequality of the system (31).

Consider the linear system

$$\dot{z}(t) = A_{22} z(t) + A_{21} u(t). \tag{32}$$

If the pair (A_{22}, A_{21}) is stabilizable then we can calculate C_2 as the optimal gain matrix such that the state-feedback law $u = -C_2 z$ minimizes the quadratic cost function:

$$J(u) = \int_0^\infty (z^T Q z + u^T R u) dt, \tag{33}$$

while verifying the Riccati equation:

$$PA_{22} + A_{22}^T P - PA_{21}R^{-1}A_{21})^T P + Q = 0, \tag{34}$$

where Q and R are matrices satisfying:

$$\begin{cases} R > 0, \\ Q \geq 0, \\ Q \text{ and } A_{22} \text{ have no unobservable mode on the imaginary axis.} \end{cases}$$

The gain matrix C_2 is then deduced by the expression:

$$C_2 = R^{-1}A_{21}^T P. \tag{35}$$

When choosing $R = I_n$,the Riccati equation and the matrix gain become:

$$\begin{aligned} &PA_{22} + A_{22}^T P = PA_{21}A_{21}^T P - Q, \\ &C_2 = A_{21}^T P. \end{aligned}$$

So the constraint $(P(A_{22} - A_{21}C_2) + (A_{22} - A_{21}C_2)^T P < 0)$ will be satisfied for whatever $Q \geq 0$. Then it will be easy to find a C_2 fulfilling the constraints (31) by adjusting the matrix parameter Q.

4 Simulation Examples

4.1 Second order bilinear system

In the case of second order homogeneous bilinear systems the state subvector x_2 is scalar and so does C_2, so all the matrices involved in the inequality system (31) are also scalar terms. Hence this latter leads to the following conditions of global stability:

$$\begin{cases} P(A_{22} - A_{21}C_2) < 0, \\ G_i H_i P(N_{i_{22}} - N_{i_{21}}C_2) \geq 0, \end{cases} \forall i = 1, ..., s, \tag{36}$$

where P is a positive scalar.

Since G_i and H do not depend on P, the problem can be reduced to the search for only one unknown variable which is C_2 such that:

$$\begin{cases} A_{22} - A_{21}C_2 < 0, \\ G_i H_i (N_{i_{22}} - N_{i21}C_2) \geq 0, \end{cases} \forall i = 1, ..., s. \tag{37}$$

Consider the second order homogeneous bilinear system defined by (2) where $m = 2$ and

$$A = \begin{pmatrix} 13 & \text{-12} \\ 10 & \text{-10} \end{pmatrix}, N_1 = \begin{pmatrix} 0.7 & 0.1 \\ 0.1 & 0.7 \end{pmatrix}, N_2 = \begin{pmatrix} \text{-2} & 0 \\ 0 & \text{-1} \end{pmatrix}.$$

In free run mode, the systems' states are divergent. The sliding mode control law is designed according to the expressions (12), (13), (6), (11). The sliding surface is defined by $S = Cx = 0$ with $C = [1\ C_2]$.

To search C_2 that guarantees the stability of the controlled system, we solve the matrix inequalities system (36) and we obtain $P = 0.1449$ and $C_2 = 2.3$.

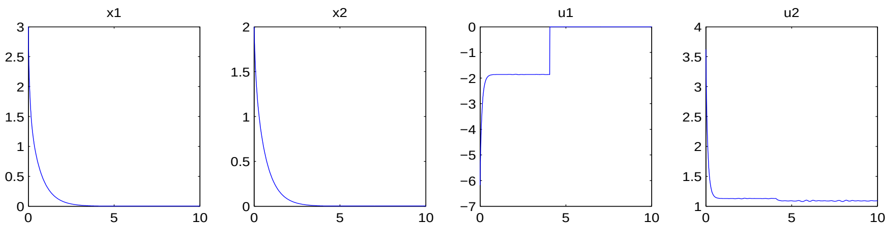

Figure 1: Second order system responses with sliding mode control, $X(0) = [3\ 2]^T$ and $\alpha = 1.01$.

When implementing the proposed control law with the sliding surface $C = [1\ \ 2.3]$ for $\alpha = 1.01$ and the initial conditions $x(0) = [3\quad 2]^T$, we obtain the simulation results presented in Figure 1. We note that the states converge to zero before 3s and with low control levels (between -8 and +4).

Even when trying to enlarge the initial conditions values or the uncertainties domains, the designed control ensure the convergence of the system's states, as shown in Figure 2.

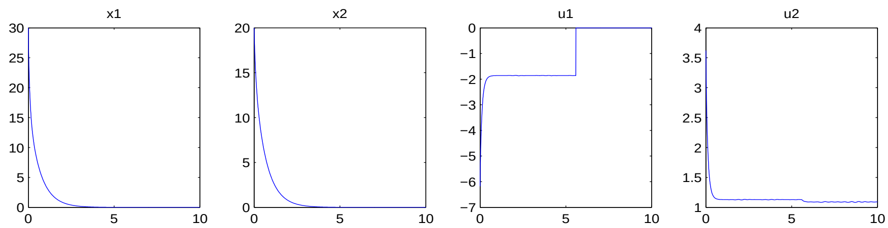

Figure 2: Second order system responses with large initial values $X(0) = [30\ 20]^T$ and $\alpha = 1.01$.

4.2 Third order bilinear system

Consider the third order bilinear system defined by (2), where $m = 1$ and

$$A = \begin{pmatrix} -2 & 3 & 1 \\ 1 & -7 & 1 \\ 2 & 1 & 0.5 \end{pmatrix},\ N = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

The resolution of the stabilization constraints system (31) gives the symmetric positive definite matrix P and the vector C_2 defined by: $P = \begin{pmatrix} 0.0777 & 0.0661 \\ 0.0661 & 0.6308 \end{pmatrix}$, $C_2 = [0.2100\quad 1.3278]$.

Simulations of the so controlled system are presented in Figures 3 and 4 respectively for small and large initial conditions, with $\alpha = 2.5$. We notice that the states converge to zero within two seconds at least. The control amplitude doesn't exceed four units.

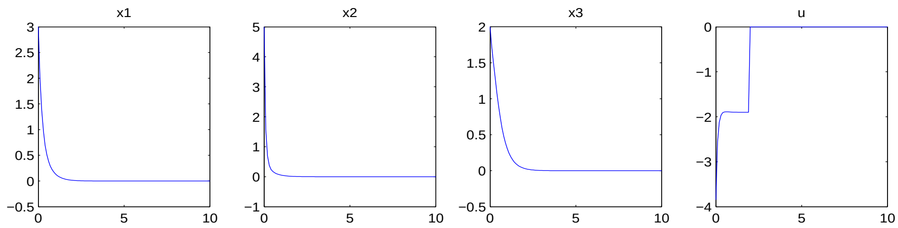

Figure 3: Third order system responses with sliding mode control, $X(0) = [3\ 5\ 2]^T$ and $\alpha = 2.5$.

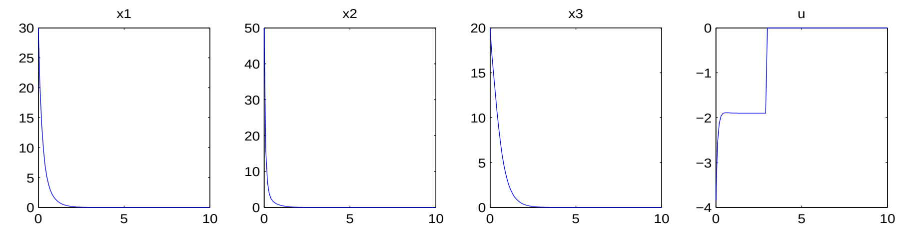

Figure 4: Third order system responses with large initial values $X(0) = [30\ 50\ 20]^T$ and $\alpha = 2.5$.

5 Conclusions

A sliding mode control approach is proposed for homogeneous bilinear systems. The control design strategy detailed in this paper enabled to provide an efficient sliding mode control constituted by two components: a switching control law basically built so as to ensure the system stability during the reaching phase, and an equivalent control law deduced from the condition of keeping the system's state quietly on the sliding surface once reached. The meticulous study held on the system's closed loop stability during this sliding phase allowed to provide sufficient conditions of global stabilization formulated in a set of linear and nonlinear matrix inequalities. The sliding surface can be automatically defined through the resolution of the stability constraints problem. Analytical and numerical cleverness have permitted to facilitate the resolution of so complex optimisation problem. In fact, for the second order systems, simplified form of the stabilization constraints is retrieved showing that the problem can be reduced to the search for only one unknown variable. On the other hand, for higher order systems the linear quadratic based algorithm suggested enables to obtain feasible solutions to the nonlinear constrained problem if there exist ones.

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