



# Approximate Controllability of a Functional Differential Equation with Deviated Argument

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**Abstract:** This paper deals with the approximate controllability of a functional differential equation with deviated argument and finite delay. Sufficient condition for approximate controllability is proved under the assumption that the linear control system is approximately controllable; thereby removing the need to assume the invertibility of a controllability operator which fails to exist in infinite dimensional space if the generated semigroup is compact. Schauder fixed point theorem is used and the  $C_0$  semigroup associated with mild solution has been replaced by the fundamental solution.

**Keywords:** *deviated argument; approximate controllability; fundamental solution; semilinear control system; delay; reachable set; Schauder fixed point theorem.*

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## 1 Introduction

In certain real world problems, delay depends not only on time but also on the unknown quantity. The differential equations with deviated arguments are generalization of delay differential equations in which the unknown quantity and its derivative appear in different values of their arguments. Functional differential equations with deviated argument model various control problems arising in the field of engineering, physics and so on. Many partial differential systems can be reduced to functional differential equations with deviated arguments, see for instance [3, 8, 15, 16]. Aftereffect, hereditary systems, equations with deviated arguments, etc. feature in several mathematical models. As a matter of fact delay differential systems are still resistant to many classical controllers.

In recent years, controllability of infinite dimensional systems has been extensively studied for various applications. The papers of Benchohra et al. [10] and Chang [19]

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