Nonlinear Dynamics and Systems Theory, **14** (3) (2014) 258–265

Existence and Multiplicity of Periodic Solutions for a Class of the Second Order Hamiltonian Systems

A. Benhassine

Department of Mathematics, Higher Institute of Informatics and Mathematics, 5000 Monastir, Tunisia

Received: October 23, 2013; Revised: July 5, 2014

Abstract: In this paper, we study the existence and multiplicity of periodic solutions of the following second-order Hamiltonian systems

$$\ddot{x}(t) + V'(t, x(t)) = 0,$$

where $t \in \mathbb{R}$, $x \in \mathbb{R}^N$ and $V \in C^1(\mathbb{R} \times \mathbb{R}^N, \mathbb{R})$. By using a symmetric mountain pass theorem, we obtain a new criterion to guarantee that second-order Hamiltonian systems has infinitely many periodic solutions. We generalize and improve recent results from the literature. Some examples are also given to illustrate our main theoretical results.

Keywords: *periodic solutions; Hamiltonian systems; mountain pass theorem; symmetric mountain pass theorem.*

Mathematics Subject Classification (2010): 34C25, 58E05, 70H05.

1 Introduction

Consider the second-order Hamiltonian systems

$$\ddot{x}(t) + V'(t, x(t)) = 0, \tag{HS}$$

where $x = (x_1, ..., x_N)$, $V \in C^1(\mathbb{R} \times \mathbb{R}^N, \mathbb{R})$ and $V'(t, x) = \nabla_x V(t, x)$. The existence and multiplicity of periodic solutions for system (HS) have been studied in many papers via critical point theory, see the classical monographs [8] and [10] and the recent papers [5, 6, 12, 13, 15, 18]. In [10], Rabinowitz established the existence of periodic solutions for (HS) under the well known Ambrosetti-Rabinowitz condition:

* Corresponding author: mailto:abderrazekbenhassine@yahoo.fr