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The Obstacle Problem Associated with Nonlinear Elliptic Equations in Generalized Sobolev Spaces

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Abstract: We prove an existence result of entropy solution to the obstacle problem associated with the equation of the type

$$-\text{div}(a(x,u,\nabla u)) + g(x,u,\nabla u) = f \in L^1(\Omega)$$

in generalized Sobolev spaces, without assuming the sign condition in the nonlinearity g via penalization methods.

Keywords: *generalized Sobolev spaces; boundary value problems; truncations; penalized equations.*

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1 Introduction

The obstacle problem is, roughly speaking, about solving a partial differential equation with the additional constraint that the solution is required to stay above a given function, the obstacle. This leads to a variational inequality. From a minimization point of view, the problem is to find a minimizer with fixed boundary value in the set of functions lying above the obstacle function. Such a set is convex and thus, a unique minimizer exists under reasonable assumptions. The balayage concept of potential theory which is the potential theoretic viewpoint of the obstacle problem is finding the smallest superharmonic function which lies above the obstacle.

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