Almost Oscillatory Three-Dimensional Dynamical Systems of First Order Delay Dynamic Equations

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Abstract: In this paper, we investigate oscillation and asymptotic properties for three dimensional systems of first order dynamic equations with delays. Most of our results are new in the discrete case.

Keywords: time scales; oscillation; three-dimensional dynamical system.


1 Introduction

In this paper, we investigate three dimensional dynamical systems with delays of the form

\[
\begin{align*}
    x^\Delta(t) &= a(t)f(y(\tau(t))), \\
    y^\Delta(t) &= b(t)g(z(\tau(t))), \\
    z^\Delta(t) &= \lambda c(t)h(x(\tau(t))),
\end{align*}
\]

(1)
on a time scale \( T \), i.e, a closed subset of real numbers, \( \tau : T \to T \) is a rd-continuous function such that \( \tau(t) < t \), \( \lim_{t \to \infty} \tau(t) = \infty \), \( \lambda = \pm 1 \), \( a, b : T \to [0, \infty) \) (not identically zero) and \( c : T \to (0, \infty) \) are rd-continuous functions such that

\[
\int_T^\infty a(s)\Delta s = \int_T^\infty b(s)\Delta s = \infty, \quad T \in T
\]

(2)
and \( f, g, h : \mathbb{R} \to \mathbb{R} \) are continuous functions satisfying

\[
uf(u) > 0, \quad ug(u) > 0, \quad \text{and} \quad uh(u) > 0 \quad \text{for} \ u \neq 0.
\]

(3)

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