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Almost Oscillatory Three-Dimensional Dynamical Systems of First Order Delay Dynamic Equations

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Abstract: In this paper, we investigate oscillation and asymptotic properties for three dimensional systems of first order dynamic equations with delays. Most of our results are new in the discrete case.

Keywords: *time scales; oscillation; three-dimensional dynamical system.*

Mathematics Subject Classification (2010): 39A10.

1 Introduction

In this paper, we investigate three dimensional dynamical systems with delays of the form

$$\begin{cases} x^{\Delta}(t) = a(t)f(y(\tau(t))), \\ y^{\Delta}(t) = b(t)g(z(\tau(t))), \\ z^{\Delta}(t) = \lambda\, c(t)h(x(\tau(t))), \end{cases} \tag{1}$$

on a time scale $\mathbb{T}$, i.e, a closed subset of real numbers, $\tau : \mathbb{T} \to \mathbb{T}$ is a rd-continuous function such that $\tau(t) < t$, $\lim\limits_{t\to\infty} \tau(t) = \infty$, $\lambda = \pm 1$, $a, b : \mathbb{T} \mapsto [0, \infty)$ (not identically zero) and $c : \mathbb{T} \mapsto (0, \infty)$ are rd-continuous functions such that

$$\int_T^{\infty} a(s)\Delta s = \int_T^{\infty} b(s)\Delta s = \infty, \quad T \in \mathbb{T} \tag{2}$$

and $f, g, h : \mathbb{R} \mapsto \mathbb{R}$ are continuous functions satisfying

$$uf(u) > 0, \quad ug(u) > 0, \quad \text{and} \quad uh(u) > 0 \quad \text{for } u \neq 0. \tag{3}$$

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