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The Structure of the Solution of Delay Differential Equations with One Unstable Positive Equilibrium

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Abstract: This paper studies the equation $\dot{x}(t) = -g(x(t)) + f(x(t-\tau))$ with one trivial equilibrium and only one unstable positive equilibrium. For a class of linear initial values, two sufficient conditions are established to guarantee that the corresponding solutions converge to the trivial equilibrium and the positive equilibrium respectively. All solutions, with the exception of two equilibria, are divided into three classes according to their eventual tendency. The first class solutions are strictly greater than 0 ultimately and converge to it; the second class ones are strictly greater than the positive equilibrium ultimately and converge to it; the third class solutions oscillate about the positive equilibrium up and down and converge to it. Furthermore, the existence of the third class of solutions is determined. Numerical simulations are given to illustrate the main results.

Keywords: *delay differential equations; convergence; oscillatory solution; attractive region; equilibrium.*

Mathematics Subject Classification (2010): 34K05; 34K60; 92B05.

1 Introduction

Delay differential equations are always the research focus of mathematicians dealing with theory of functional differential equations and scientists applying the theory to practical problems. It is not difficult to found a variety of application of delay differential equation in several fields of natural science such as viscoelasticity, mechanics, models for nuclear reactors, distributed networks, heat flow, neural networks, combustion theory, interaction of species, microbiology, learning models, epidemiology, physiology see e.g. [9, 11, 15, 22].

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